Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2024

Subject Code:3140708 **Date:01-07-2024**

Subject Name: Discrete Mathematics

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Define injective function. Given $A = \{2, 5, 6\}$, $B = \{3, 4, 2\}$, find $(A - B)$ and $(B - A)$.	**03**
	(b)	Determine the relation $\leq$ (less than or equal) on the set $\mathbb{Z}$ of integers are reflexive, symmetric, anti-symmetric, transitive.	**04**
	(c)	(i) Check whether the function $f(x) = x^3 - 2$, for $x \in \mathbb{R}$ is invertible function. If so, find $f^{-1}(x)$.	**03**
		(ii) Prove that a tree with n vertices has $n - 1$ edges.	**04**
Q.2	**(a)**	Identify the statement $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is tautology or contradiction without constructing the truth table.	**03**
	(b)	Let G be the subset of 2×2 real matrices with a nonzero determinant. Check whether G is group under matrix multiplication. If so, is it abelian group?	**04**
	(c)	(i) Symbolize the expression “John is a bachelor and this painting is red”.	**03**
		(ii) Express the following using predicate, quantifier and logical connectives. Also verify the validity of the consequence. Everyone who graduates gets a job. Ram is graduated. Therefore, Ram got a job.	**04**
		OR	
	(c)	Use a truth table to determine whether the following argument form is valid. $p \rightarrow q$ $p \rightarrow r$ $\therefore p \rightarrow q \vee r$	**07**
Q.3	**(a)**	Let g be a homomorphism from a group $(G,*)$ to a group (H,Δ). Show that $g(e_G) = e_H$ and for any $a \in G$, $g(a^{-1}) = (g(a))^{-1}$.	**03**
	(b)	Prove that : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.	**04**
	(c)	(i) Prove that every cyclic group is abelian.	**03**
		(ii) Consider the set of positive integers $\mathbb{N}$. Check which of $(\mathbb{N}, +)$ and $(\mathbb{N},\times)$ are semigroup and which are monoid?	**04**
		OR	

Q.3 **(a)** Find left cosets and right cosets of $H = \{0,3\}$ in the group $(\mathbb{Z}_6, +_6)$. **03**

(b) (i) Suppose repetitions are not allowed, how many four digit numbers can be formed from six digits 1,2,3,5,7,8? **04**
(ii) How many of such numbers less than 4000?
(iii) How many in (i) are even?
(iv) How many in (i) are divisible by 10?

(c) Show that $(R, +, \times)$ is an integral domain, where $R = \{a + b\sqrt{5} \,/\, a,\ b\ \in\ \mathbb{Z}\}$. **07**

Q.4 **(a)** Let $S = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$. Draw the graph of R and hence write partition of S. **03**

(b) Define Lattice. Draw the Hasse diagram of (S_{12}, D), where D is the relation of "division" in $\mathbb{N}$ such that for any $a, b \in \mathbb{N}$, aDb iff a divides b and S_{12} is the set of all divisors of 12. **04**

(c) Let $\langle L, \leq \rangle$ be a lattice. Show that for $a, b, c\ \in\ L$, following inequalities holds. **07**
(i) $a \oplus (b\ *\ c)\ \leq\ (a \oplus b)\ *\ (a \oplus c)$ and
(ii) $a\ *\ (b \oplus c) \geq\ (a\ *\ b) \oplus (a\ *\ c)$.

OR

Q.4 **(a)** Let the POSET $(\rho(A), \leq)$ where $A = \{a, b, c\}$, relation is subset. Find **03**
(i) Upper bound of $\{\{\ \}, \{a\}, \{c\}\}$,
(ii) GLB of $\{\{\ \}, \{a\}, \{c\}\}$, if exist,
(ii) LUB of $\{\{\ \}, \{a\}, \{c\}\}$, if exist.

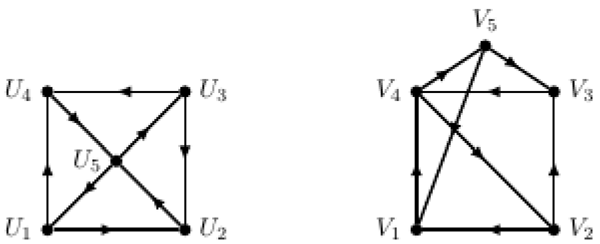
(b) Solve $a_n =\ a_{n-1}\ +\ a_{n-2}, a_0 =\ 0,\ a_1 =\ 1$. **04**

(c) Given the relation matrices M_R and M_S, find $M_{R\circ S}$, $M_{\tilde{R}}$, $M_{\tilde{S}}$, $M_{\widetilde{R\circ S}}$, and show that $M_{\widetilde{R\circ S}} = M_{\tilde{S}} \circ M_{\tilde{R}}$. **07**

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

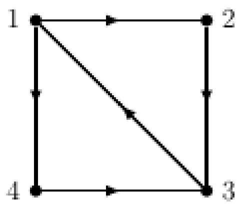
Q.5 **(a)** Define Isolated node, Binary tree and Regular graph. **03**

(b) Check whether the following graphs are isomorphic or not. **04**



$G(V, E)$ $H(V, E)$

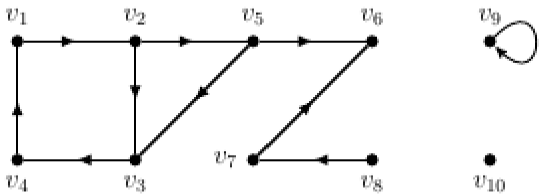
(c) Define path matrix. Warshall's algorithm to obtain path matrix from the adjacency matrix of following graph **07**



OR

Q.5 **(a)** A graph G has 15 edges, 3 vertices of degree 4 and other vertices of degree 3. Find the number of vertices in G. **03**

(b) Find all the node base of the given digraph. Also find $d(V_3, V_6), d(V_6, V_3)$. **04**



(c) Draw binary trees whose post-order produced the string d-e-c-g-j-h-f-b-l-n-q-r-p-m-k-a and pre-order produced the string a-b-d-h-e-i-j-c-f-g-k and in-order produced the string h-d-b-i-e-j-a-f-c-k-g. **07**

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