Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2024

Subject Code:3130006 Date:16-07-2024

Subject Name: Probability and Statistics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

MARKS

Q.1 **(a)** Define and give the example of Random variable. **03**

(b) What is the probability that a leap year selected at random will have 53 Sundays? **04**

(c) In a bolt factory, three machines A, B and C produce 25%, 35% and 40% of total output respectively. It was found that 5%,4% and 2% are defective bolts in the production by machines A, B, C respectively. A bolt is chosen at random from the total output and is found to be defective. Find the probability that it is manufactured from (i) Machine A(ii) Machine B (iii) Machine C. **07**

Q.2 **(a)** A bag contains 3 red and 4 white balls. Two draws are made without replacement. What is the probability that both balls are red. **03**

(b) The probability that a student A solves a mathematics problem is $\frac{2}{5}$ and the probability that a student B solves it is $\frac{2}{3}$. What is the probability that (i) the problem is not solved (ii) both A and B can solve the problem, working independently of each other? **04**

(c) Verify that the following function $F(x)$ is a distribution function. **07**

$$F(x) = \begin{cases} 0; & x < 0 \\ 1 - e^{-\frac{x}{4}}; & x \geq 0 \end{cases}$$

Also, find the probabilities $P(X \leq 4), P(X \geq 8),\ P(4 \leq X \leq 8)$.

OR

(c) The probability mass function of a random variable X is zero except at the points $X = 0, 1, 2$. At these points, it has the values $P(X = 0) = 3c^3,\ P(X = 1) = 4c - 10c^2, P(X = 2) = 5c - 1$. Find (i) c (ii) $P(X < 1)$ (iii) $P(1 < X \leq 2)$ (iv) $P(0 < X \leq 2)$. **07**

Q.3 **(a)** Find the constant k such that the function **03**

$$f(x) = \begin{cases} kx^2; & 0 < x < 3 \\ 0; & otherwise \end{cases}$$

is a probability density function,

(b) A random variable X has the following distribution: **04**

X	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Find (i) mean (ii) variance.

(c) Compute Karl Pearson's coefficient of correlation between X and Y for the following data: **07**

x	10	14	18	22	26	30
y	18	12	24	6	30	36

OR

Q.3 **(a)** Find coefficient of correlation between x and y if the regression lines are: $x + 6y = 6$ and $3x + 2y = 10$. **03**

(b) Calculate the first four moments from the following data: **04**

x	0	1	2	3	4	5	6	7	8
y	5	10	15	20	25	20	15	10	5

(c) The following data give the experience of machine operators and their performance rating as given by the number of good parts turned out per 100 piece. **07**

Operator	1	2	3	4	5	6
Performance rating (x)	23	43	53	63	73	83
Experience (y)	5	6	7	8	9	10

Calculate the regression line of performance rating on experience and also estimate the probable performance if an operator has 11 years of experience.

Q.4 **(a)** Explain the term related to testing of hypothesis: (i) Null hypothesis (ii) Alternate hypothesis and (iii) Level of Significance **03**

(b) A dice is tossed 960 times and it falls with 5 upwards 184 times. Is the dice unbiased at a level of significance of 0.01? $(|Z_{0.01}| = 2.58)$ **04**

(c) A set of five similar coins is tossed 320 times and result is obtained as follows **07**

No of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. (Critical value $\chi^2_{0.05} = 11.07$)

OR

Q.4 **(a)** The heights of 10 males of a given locality are found to be 175, 168, 155, 170, 152, 170,175,160,160 and 165 cm. Based on this sample, find the 95% confidence limits for the heights of males in that locality. $[t_{0.05}(v = 9) = 2.262]$ **03**

(b) Random samples drawn from two countries gave the following data relating to the heights of adult males: **04**

	Country A	Country B
Standard deviation (in inches)	2.58	2.50
Number in samples	1000	1200

Is the difference between the standard deviation significant? $(|Z_{0.05}| = 1.96)$

(c) If a random variable has a Poisson distribution such that $P(X = 1) = P(X = 2)$, find (i) the mean of the distribution (ii) $P(X = 4)$ (iii) $P(X \geq 1)$ (iv) $P(1 < X < 4)$. **07**

Q.5 **(a)** Fit a straight line $y = ax + b$ to the following data: **03**

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

(b) Fit a curve $y = ab^x$ to the following data: **04**

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

(c) The lifetime of a certain kind of batteries has a mean life of 400 hours and the standard deviation as 45 hours. Assuming the distribution of lifetime to be normal, find (i) the percentage of batteries with a lifetime of at least 470 hours (ii) the proportion of batteries with a lifetime between 385 and 415 hours, and (iii) the minimum life of the best 5% of batteries. $(P(0 < z < 0.33) = 0.1293)$ **07**

OR

Q.5 **(a)** The mean and variance of a binomial variate are 8 and 6. Find $P(X \geq 2)$. **03**

(b) Out of 800 families with 5 children each, how many would you except to have (i) 3 boys? (ii) 5 girls? **04**

(c) Fit a second-degree parabolic curve to the following data: **07**

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

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