Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2024

Subject Code: 3130006 Date:21-11-2024

Subject Name: Probability and Statistics

Time: 10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**
5. **Use of log Table/Statistical Table/Any required Table are permitted.**

MARKS

Q.1 **(a)** Define and give the example of Mutually exclusive events. **03**

(b) One card is drawn at random from a pack of 52 cards. Find the probability of getting a king or a red card. **04**

(c) Data on the readership of a certain magazine show that the proportion of male readers under 35 is 0.40 and that over 35 is 0.20. If the proportion of readers under 35 is 0.70, find the probability of subscribers that are female over 35 years. Also, calculate the probability that a randomly selected male subscriber is under 35 years of age. **07**

Q.2 **(a)** Three students A, B, C are in a running race. If $P(A) = P(B) = 2P(C)$, then find the probability that B or$C$ wins. **03**

(b) Let X be a continuous random variable with pdf **04**

$$f(x) = kx(1-x), 0 \le x \le 1$$

Find k and determine $P(0 \le X \le 0.5)$.

(c) The contents of urns I, II and III are as follows: **07**

1 white, 2 red, and 3 black balls
2 white, 3 red and 1 black ball, and
3 white, 1 red and 2 black balls.

One urn is chosen at random and two balls are drawn. They happen to be white and red. Find the probability that they came from (i) Urn I (ii) Urn II (iii) Urn III.

OR

(c) A random variable X has the following probability distribution: **07**

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	$3k$

(i) Find the value of k
(ii) Find $P(x \ge 2)$
(iii) $P(-2 < x < 2)$

Q.3 **(a)** Let the continuous random variable X have the probability density function $f(x) = \begin{cases} \frac{1}{x^4}; & 1 < x < \infty \\ 0; & otherwise \end{cases}$ **03**

Find distribution function $F(x)$.

(b) A random variable X has the following distribution: **04**

X	-3	-2	-1	0	1	2
$P(X = x)$	0.05	0.1	0.2	0.3	0.2	0.15

Find (i) $E(X)$ (ii)$Var(X)$.

(c) Compute Karl Pearson's coefficient of correlation between X and Y for the following data: **07**

x	2	4	5	6	8	11
y	18	12	10	8	7	5

OR

Q.3 **(a)** If the two lines of regression are $2x - 5y + 30 = 0$ and $10x - 4y - 104 = 0$ which of these are lines of regression of x on y and y on x? **03**

(b) Calculate the first four moments from the following data: **04**

x	5	10	15	20	25
f	6	10	14	6	4

(c) Obtain the line of regression of monthly sales (y) on advertisement expenditure (x) and estimate the monthly sales when the company will spend Rs. 50000 on advertisement, if the data on y and x are as follows: **07**

y (in lac)	74	76	60	68	79	70	71	94
x (in thousand)	43	44	36	38	47	40	41	54

Q.4 **(a)** Explain the term related to testing of hypothesis: (i) One Tailed Test (ii) Two Tailed Test (iii) Critical Region. **03**

(b) A coinwas tossed 960 times and returned heads 183 times. Test the hypothesis that the coin is unbiased.Usea 0.05 level of significance? $(|Z_{0.05}| = 1.96)$ **04**

(c) In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys has the same defect. Is a difference between the proportions significant at 0.05 level of significance? $(|Z_{0.05}| = 1.96)$ **07**

OR

Q.4 **(a)** The mean breaking strength of a rope is 160 kgs. If 6 bricks (selected from different piles) have a mean breaking strength of 154.3 kgs with a standard deviation of 6.4 kgs. Test the null hypothesis $\mu = 160$ kgs. against the alternative hypothesis $\mu < 160$ kgs. at 1% level of significance. $[t_{0.01}(v = 5) = 3.365]$ **03**

(b) Examine whether the two samples for which the data are given in the following table could have been drawn from populations with the same SD. **04**

	Size	SD
Sample I	100	5
Sample II	200	7

$(|Z_{0.05}| = 1.96)$

(c) If X is a Poisson variate such that $P(X = 0) = P(X = 1)$, find $P(X = 0)$ and using recurrence formula, find the probabilities at $x = 1,2,3,4$ and 5. **07**

Q.5 **(a)** Using least squares approximation method, fit a straight line$y = ax + b$ to the following data: **03**

x	1	2	3	4	5
y	14	27	40	55	68

(b) Using least squares approximation method,fit a curve $y = ae^{bx}$ to the following data: **04**

x	1	5	7	9	12
y	10	15	12	15	21

(c) Distribution of height of 1000 students is normal with mean 165 cms and standard deviation 15 cms. How many soldiers are of height (i) less than 138 cms. (ii) more than 198 cms. (iii) between 138 and 198 cms. $(P(z = 1.8) = 0.4641, P(z = 2.2) = 0.4841)$ **07**

OR

Q.5 **(a)** In a throw of a dice the appearance of a number 4 is taken as a success. If the dice is thrown 5 times, find mean, variance and standard deviation. **03**

(b) If a random variable has a Poisson distribution such that $P(X = 1) = P(X = 2)$, find (i) the mean of distribution, (ii) $P(X = 4)$ (iii) $P(X \geq 1)$ (iv) $P(1 < X < 4)$. **04**

(c) Using least squares approximation method, fit a second-degree parabolic curve to the following data: **07**

X	1	2	3	4	5
Y	5	12	26	60	97

Also, estimate y at $x = 6$.

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