GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II EXAMINATION – WINTER 2024

Subject Code:3110015   **Date:10-01-2025**

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM   **Total Marks:70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Find a such that $(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal.	**03**
	(b)	Solve $ye^x\,dx+(2y+e^x)dy=0$	**04**
	(c)	Verify Green's theorem in a plane for the integral $\int_C\left[(x-2y)dx+x\,dy\right]$ taken around the circle $x^2+y^2=4$.	**07**

Q.2	**(a)**	Find the Laplace transform of $(\sin 2t-\cos 2t)^2$.	**03**
	(b)	Find the Fourier sine integral of $f(x)=e^{-bx}$.	**04**
	(c)	Using the Frobenius method, obtain the series solution for $2x(1-x)y''+(1-x)y'+3y=0$ about $x_0=0$.	**07**

OR

	(c)	Using the method of undetermined coefficients, solve $(D^2-9)y=x+e^{2x}-\sin 2x$.	**07**

Q.3	**(a)**	Find the arc length of the curve $\vec{r}(t)=t^2\hat{i}+t^3\hat{j}$ between (1,1) and (4,8).	**03**
	(b)	Find the Laplace transform of $\dfrac{e^{-t}\sin t}{t}$.	**04**
	(c)	State the convolution theorem and verify it for $f(t)=t$ and $g(t)=e^{2t}$.	**07**

OR

Q.3	**(a)**	Find the inverse Laplace transform of $\tan^{-1}s$.	**03**
	(b)	Solve $x^2p^2+3xyp+2y^2=0$.	**04**
	(c)	Solve the initial value problem using Laplace transformation $y''-3y'+2y=4t$ with $y(0)=1,\ y'(0)=-1$.	**07**

Q.4	**(a)**	Find the inverse Laplace transform of $\dfrac{e^{-\pi s}}{s^2-2s+2}$.	**03**
	(b)	Solve $(D^2+1)y=e^{-x}$.	**04**
	(c)	Using method of variation of parameters, solve $(D^2-2D+2)y=e^x\tan x$.	**07**

OR

Q.4	**(a)**	Classify the singular points of the equation $x^3(x-2)y''+x^3y'+6y=0$.	**03**

(b) Find the Laplace transform of $\sin \sqrt{t}$.   **04**

(c) Find the series solution of $\left(1+x^2\right)y'' + xy' - 9y = 0$.   **07**

Q.5 **(a)** Solve $dr + \left(2r\cot\theta + \sin 2\theta\right)d\theta = 0$ **03**

(b) If $y_1 = \dfrac{\sin x}{x}$ is one of the solution of $xy'' + 2y' + xy = 0$, find the second solution.   **04**

(c) Show that (i) $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\sin x$ (ii) $J_{-\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\cos x$.   **07**

OR

Q.5 **(a)** Find the Laplace transform of $\displaystyle\int_0^t \int_o^t \sin at\, dt\, dt$.   **03**

(b) Solve $\left(x^2 D^2 - xD + 2\right)y = 6$.   **04**

(c) Prove that $J_0^2(x) + 2\left[J_1^2(x) + J_2^2(x) + J_3^2(x) + \ldots\right] = 1$ **07**

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