

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2023****Subject Code:3144007****Date:24-01-2024****Subject Name: Numerical and Statistical Methods****Time: 10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.
5. Statistical Tables are to be provided as and when necessary.

Q.1 (a) There are 5 green and 7 red balls. Two balls are selected one by one without replacement. Find the probability that the first is green and the second is red. **03**

(b) Find the positive root of the equation $4x = e^x$ using Regula Falsi method and correct to 3 decimal places. **04**

(c) Calculate $f(1925)$ using suitable interpolation formula: **07**

x	1891	1901	1911	1921	1931
f(x)	46	66	81	93	101

Q.2 (a) The probability of a student passing in science is $\frac{4}{5}$ and that of the student passing in both science and math is $\frac{1}{2}$. What is the probability of that student passing in math knowing that he passed in science? **03**

(b) A random variable is defined as follows: $P(X=1) = p$, $P(X=0) = 1 - p$, $0 < p < 1$. Find the expectation and variance of the distribution. **04**

(c) Bag-I contains 3 red and 4 black balls while another Bag-II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag-II. **07**

OR

(c) Using Lagrange's interpolation formula, find $y(10)$ from the following table: **07**

x	5	6	9	11
y	12	13	14	16

Q.3 (a) Calculate the median from the following data: **03**

Observation	8-10	10-12	12-14	14-16
Frequency	4	8	12	16

(b) Use Simpson's $1/3^{\text{rd}}$ rule to evaluate $\int_0^2 \sqrt{1+e^x} dx$ by taking $n = 4$ **04**

(c) The average number of acres burned by forest and range fires in a large district is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year? **07**

OR

- Q.3 (a)** A company receives an average of 2.5 calls per day in a particular period. Find the probability that on a certain day, the company receives exactly 4 calls during the same period assuming Poisson distribution. **03**
- (b)** Two judges ranked 10 participants in an art competition in the following way: **04**
- | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|----|
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Y | 6 | 2 | 9 | 7 | 1 | 4 | 8 | 3 | 10 | 5 |
- Find the Spearman's rank correlation coefficient.
- (c)** Find $y(0.2)$ using Taylor's series method for the differential equation $y' = x^2y - 1, y(0) = 1$, taking step length 0.1. **07**
- Q.4 (a)** The mean length of the lumber is supposed to be 8.5 feet. A builder wants to check whether the shipment of lumber she receives has a mean length different from 8.5 feet. If the builder observes that the sample mean of 61 pieces of lumber is 8.3 feet with a sample standard deviation of 1.2 feet, what will she conclude at 1% significant level? **03**
- (b)** Solve $x - 2 \sin x - 3 = 0$ by Newton Raphson method correct up to three significant digits. **04**
- (c)** Fit a straight line from the given data: **07**
- | | | | | | |
|---|----|----|----|----|----|
| x | 5 | 10 | 15 | 20 | 25 |
| y | 16 | 19 | 23 | 26 | 30 |
- OR**
- Q.4 (a)** Find the coefficient of correlation between the X and Y variables from the following two regression equations: $2y - x = 50$ and $3y - 2x = 10$. **03**
- (b)** A soft drink maker claims that the majority of adults prefer its leading beverage over that of its main competitor's. To test this claim, 500 randomly selected people were given the two beverages. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Test at 5% level of significance to support the soft drink maker's claim. **04**
- (c)** A die is tested to check if it is biased or not. The results are given in the following table: **07**
- | | | | | | | |
|-----------|---|----|----|---|----|----|
| Score | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 7 | 10 | 11 | 9 | 12 | 10 |
- Use χ^2 test at 1% significant level to check if the die is fair.
- Q.5 (a)** The following table gives the values of mean and variance of heights and weights of the 10th standard students at a school. **03**
- | | | |
|----------|--------------|--------------|
| | Height (cm.) | Weight (kg.) |
| Mean | 155 | 46.5 |
| Variance | 72.25 | 28.09 |
- State which one is more variable?
- (b)** A random sample of 15 pairs of observations from a bivariate normal population gives a correlation coefficient $r = 0.25$. Is it likely that the variables are uncorrelated in the population? Test at 5% significance level) **04**

- (c) Find the coefficient of skewness: 07

Observations	5	10	15	20	25	30
Frequency	3	8	12	15	18	14

OR

- Q.5 (a)** A continuous random variable has the probability density function given by $f(x) = ax$, $0 \leq x \leq 1$. Determine the constant a and also find $P(X \leq 1/2)$. 03

- (b)** A toy manufacturer wants to get batteries for toys. A team collected 28 samples from supplier A and the variance was 38 hours. The team also collected 25 samples from supplier B and the variance was 83 hours. At 5% significance level, determine if there is a difference in the variances. 04

- (c)** Apply Runge-Kutta method of fourth order to calculate $y(0.2)$, where $\frac{dy}{dx} = \frac{x-y}{2}$, $y(0) = 1$ taking $h = 0.1$. 07
