

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III (NEW) EXAMINATION – WINTER 2023****Subject Code:3130006****Date:12-01-2024****Subject Name:Probability and Statistics****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Define and give examples of Random Experiment and Sample space. **03**

(b) For a certain model car, the probability of the air conditioner failing before the warranty expires is 0.36, the probability of alternator failing is 0.26, and the probability of both failing is 0.11. Find the probability of the air conditioner or the alternator failing before the warranty expires. **04**

(c) A microchip company has three machines that produce the chips. Machine- I produces 45% of the chips, but 5% of its chips are defective. Machine-II produces 35% of the chips and 10% of its chips are defective. Machine-III produces the rest of the chips and 2% of its chips are defective. Find the probability of:

A randomly selected chip is found to be

- (i) a defective chip that has come from Machine-I,
- (ii) a non-defective chip that has come from Machine-II,
- (iii) a defective chip that has come from Machine-III.

Q.2 (a) Define Exponential Distribution and Gamma distribution. **03**

(b) The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week. (Use χ^2 at 5% level of significance for 6 degree of freedom is 12.59). **04**

Days	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
No. of Accidents	14	16	8	12	11	9	14

(c) Fit a second-degree parabola for the following data: **07**

x:	0	1	2	3	4
y:	3	6	11	18	27

Estimate the value of y when x = 5.

OR

(c) Fit a relation of the form $y = ab^x$ for the following data by the method of least squares: **07**

x:	2	3	4	5	6
y:	8.3	15.4	33.1	65.2	126.4

Estimate the value of y when x = 1.

Q.3 (a) An insurance company has discovered that only 0.1% of the population is involved in a certain type of accident every year. If its 1000 policy-holders are selected at random **03**

from the population, what is the probability that not more than 2 of its clients are involved in such accident next year?

- (b) Suppose that the current measurements in a strip of wire are assumed to follow a normal distribution with a mean of 10 milliamperes and variance of 4 (milliamperes)². What is the probability that a measurement (i) exceeds 13 milliamperes, (ii) is at most 10 milliamperes, (iii) between 10 and 13 milliamperes? [Use: $P(Z \leq 1.5) = 0.9332$]. **04**
- (c) Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in next 10 samples, (i) exactly 2, (ii) at least 4, (iii) between 2 and 7, samples contain the pollutant. **07**

OR

- Q.3** (a) Form the binomial distribution for the experiment of tossing a coin three times and counting the number of heads appear. **03**
- (b) In a normal distribution 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **04**
[Use: $P(0 < Z < 1.405) = 0.42$, and $P(0 < Z < 0.495) = 0.19$].
- (c) For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with mean of 2.3 flaws per millimeter. Determine the probability of (i) exactly 2 flaws in 1 millimeter of wire, (ii) at least 1 flaw in 2 millimeters of wire, (iii) between 2 and 5 flaws in 1 millimeter of wire. **07**

- Q.4** (a) For the following data, which of the series A and B shows greater variation? **03**

Series	Mean	Standard Deviation
A	160	10
B	60	5

- (b) In a partially, destroyed record on analysis of correlation data, only the following are legible: Variance of x , $\sigma_x^2 = 9$, regression equation $8x - 10y + 66 = 0$, $40x - 18y = 214$. Find (i) mean values of x and y , (ii) the standard deviation of y . **04**
- (c) Ten competitors in a contest are ranked by three judges in the following order: **07**

1 st Judge	1	6	5	10	3	2	4	9	7	8
2 nd Judge	3	5	8	4	7	10	2	1	6	9
3 rd Judge	6	4	9	8	1	2	3	10	5	7

Use the correlation coefficient to determine which pair of judges has the nearest approach.

OR

- Q.4** (a) Calculate Pearson's coefficient of skewness for the following data: **03**

x:	12	17	22	27	32
f:	28	42	54	108	129

- (b) The number of messages sent per hour over a computer network has the following probability distribution: **04**

X = No. of messages	10	11	12	13	14	15
P(x)	0.08	0.15	0.30	0.20	0.20	0.07

Determine the mean and standard deviation of the number of messages sent per hour.

- (c) Calculate the coefficient of correlation between x and y for the following data: **07**

x:	65	66	67	67	68	69	70	72
y:	67	68	65	68	72	72	69	71

- Q.5 (a)** Define Statistical Hypothesis and types of errors occurring while testing of hypothesis. **03**

- (b) To test whether a particular training improved the performance, a similar training was given to 8 participants, their scores both before and after the training are given below: **04**

Score (Before):	44	40	61	52	32	44	70	41
Score (After):	53	38	69	57	46	39	73	48

Test at 5% level of significance if the training was effective in terms of performance on the test. [Use $t_{7,0.05} = 2.37$].

- (c) A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is 130°F. A sample of size $n = 9$ systems, when tested, yields a sample average activation temperature of 131.08°F. If the distribution of activation times is normal with standard deviation 1.5°F, does the data contradict the manufacturer's claim at 1% level of significance. [Use $Z_{0.01} = 2.58$.] **07**

OR

- Q.5 (a)** A random sample of size 20 from a normal population has mean 40 and standard deviation of 4. Test the hypothesis that the population mean is 45. [Use $t_{0.05,19} = 2.09$.] **03**

- (b) Analysis of a random sample consisting of size $m = 20$ specimens of cold-rolled steel to determine yield strengths resulted in a sample average strength of $\bar{x} = 29.8$ ksi. A second random sample of size $n = 25$ two-sided galvanized steel specimens gave a sample average strength of $\bar{y} = 34.7$ ksi. Assuming that the two yield-strength distributions are normal with $\sigma_1 = 4.0$ and $\sigma_2 = 5.0$, does the data indicate that the corresponding true average yield strengths μ_1 and μ_2 are different? Use 5% level of significance. [Use $Z_{0.05} = 1.96$.] **04**

- (c) Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxide layers is a critical characteristic of the wafer, and low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Twenty wafers are etched in each gas. The sample standard deviations of oxide thickness are $S_1 = 1.96$ angstroms and $S_2 = 2.13$ angstroms, respectively. Is there any evidence to indicate that either gas is preferable? [Use $F_{0.05,(19,19)} = 2.16$.] **07**
