Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I (NEW) EXAMINATION – WINTER 2023

Subject Code:3110015 Date:23-01-2024

Subject Name: Mathematics - 2

Time: 02:30 PM TO 05:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

MARKS

Q.1 (a) Define Solenoidal Vector field. Find the constant a such that the vector $(x+3)\ \hat{\imath}+(y-2z)\hat{\jmath}+(x+az)\hat{k}$ is solenoidal. **03**

(b) If $\bar{F}=3x^2y\,\hat{\imath}+(x^3-2yz^2)\hat{\jmath}+(3z^2-2y^2z)\hat{k}$, is conservative, then find **04**

(i) its scalar potential ϕ.
(ii) the work done in moving from (2,1,1) to (3,0,1).

(c) Verify Green's theorem in the plane for $\oint_C (xy^2-2xy)dx+(x^2y+3)\,dy$, where C is the region bounded by the rectangle with vertices $(-1,0),(1,0),(1,1)$ and $(-1,1)$. **07**

Q.2 (a) State first shifting theorem for Laplace transform. **03**

Evaluate $(i)\ L[e^{-t}t^5]$ $(ii)\ L^{-1}\left[\frac{1}{s^2+2s+2}\right]$.

(b) Find (i) $L\left[\frac{1-\cos 2t}{t}\right]$ (ii) $L^{-1}\left[\frac{54}{(s^2+9)(s^2-9)}\right]$. **04**

(c) Using Laplace transform solve the initial value problem: **07**

$y''-y=t$, with $y(0)=1, y'(0)=-1$.

OR

(c) Find the Fourier sine integral of $f(x)=e^{-bx}$ and show that **07**

$$\frac{\pi}{2}\ e^{-bx}=\int_0^\infty \frac{\lambda\sin\lambda x}{\lambda^2+b^2}\,d\lambda.$$

Q.3 (a) Define linear differential equation. Solve: $\frac{dy}{dx}+y=e^{-x}$, given that $y(1)=1$. **03**

(b) Solve: (i) $(D^2+7D-18)y=0$, (ii) $y''+y'+2y=0$. **04**

(c) (i) Solve: $(x+y-1)dx+(2x+2y-3)dy=0$, **07**
(ii) Solve: $x^2\,p^2-4y^2=0$.

OR

Q.3 (a) Solve: $y^2\frac{dx}{dy}+yx=y^3x^2$. **03**

(b) Determine a second solution of $x^2\frac{d^2y}{dx^2}-2y=0, x>0$, where first solution is $y_1=\frac{1}{x}$. **04**

(c) (i) Solve: $p = \sin(y - px)$, **07**
(ii) Solve: $(x^3 + y^3)dx - xy^2dy = 0$.

Q.4 **(a)** Solve: $(D^2 + 3D + 2)y = \sin 2x$. **03**

(b) Solve: $x^3y''' + x^2y'' = x^3$. **04**

(c) Using the method of undetermined coefficients solve $y'' + 9y = 2x^2$. **07**

OR

Q.4 **(a)** Solve: $(D^2 + 4D)y = x + x^2$. **03**

(b) Solve: $(D^2 - 6D + 9)y = x^2e^{3x}$. **04**

(c) Using the method of variation of parameters solve: $y'' - 2y' + y = xe^x \sin x$. **07**

Q.5 **(a)** Determine the singular points of the differential equation $2x(x-2)^2\, y'' + 3xy' + (x-2)y = 0$ and classify them as regular or irregular. **03**

(b) Find the series solution of $y'' - y' = 0$. **04**

(c) Solve the differential equation $3xy'' - (x-2)y' + 2y = 0$ by using Frobenius method. **07**

OR

Q.5 **(a)** Discuss about the ordinary points, regular singular points and irregular singular points for the differential equation $x^3y'' + 5xy' + 3y = 0$. **03**

(b) Find the power series solution of $y' + 2xy = 0$ in powers of x. **04**

(c) (i) Express $J_3(x)$ in terms of $J_1(x)$ and $J_2(x)$. **07**
(ii) Express $f(x) = 4x^3 + 6x^2 + 7x + 1$ in terms of Legendre's polynomials.

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