Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–IV(NEW) EXAMINATION – WINTER 2022

Subject Code:3140708   **Date:16-12-2022**
Subject Name:Discrete Mathematics
Time:10:30 AM TO 01:00 PM   **Total Marks:70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

		Marks

Q.1 **(a)** A committee 5 persons, is to be formed from 6 men and 4 women. In how many ways this can be done when (i) at least 2 women are included, (ii) at most 2 women are included.   **03**

(b) If $A = \{4,5,7,8,10\}$, $B = \{4,5,9\}$ and $C = \{1,4,6,9\}$, then verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.   **04**

(c) Define Functionally complete set of connectives, Principal Disjunctive Normal Form (PDNF). Obtain PDNF for the expression $\neg((p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r))$ **07**

Q.2 **(a)** Define Partial Order Relation. Illustrate with an example.   **03**

(b) Define one – one function. Show that the function $f : R \to R$, $f(x) = 3x - 7$ is one – one and onto both. Also find its inverse.   **04**

(c) Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$ using method of undetermined coefficients.   **07**

OR

(c) Use generating function to solve a recurrence relation $a_n = 3a_{n-1} + 2$ with $a_0 = 1$.   **07**

Q.3 **(a)** Define Partition of a Set. Let $A = \{1,2,3,4,5\}$ and $R = \{(1,2),(1,1),(2,1),(2,2),(3,3),(4,4),(5,5)\}$ be an equivalence relation on A. Determine the partition for R^{-1}, if it an equivalence relation.   **03**

(b) Draw Hasse Diagram for the lattice (S_{30}, D) where S_{30} is the set of divisors of 30 and D is the relation divides.   **04**

(c) Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ where $a,b \in R$ is a field with respect to matrix addition and matrix multiplication.   **07**

OR

Q.3 **(a)** Define Semi Group, Monoid. Give an example of an algebraic structure which is semi group but not monoid.   **03**

(b) Consider the a relation R defined on $A = \{1,2,3\}$ whose matrix representation is given below. Determine its inverse R^{-1} and complement R'. **04**

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Define free variable and bound variable. **07**
Rewrite the following argument using quantifiers, variables and predicate symbols. Prove the validity of the argument.

"All healthy people eat an apple a day.
Ram does not eat apple a day.
Ram is not a healthy person."

Q.4 (a) Define Abelian group and prove that the set $\{0,1,2,3,4\}$ is a finite abelian group under addition modulo 5 as binary operation. **03**

(b) Define even permutation. Show that the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 4 & 1 & 3 \end{pmatrix}$ is odd and $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 4 & 5 & 2 & 1 \end{pmatrix}$ is even. **04**

(c) Define Principal Ideal. Find all the principal Ideal of the ring $\left[\{0,1,2,3,4,5\}, +_6, \times_6\right]$. **07**

OR

Q.4 (a) Define cut vertex. List out all the cut vertices of the graph given in Figure 1. **03**

(b) Define adjacency matrix and path matrix of a graph. Find out adjacency matrix for the graph given in Figure 1. **04**

(c) Define the terms: Simple Graph, Multi – Graph, Weighted Graph, Degree of a vertex, in degree and out degree of a vertex. Illustrate each with an example. **07**

Q.5 (a) State Lagrange's theorem and find out all possible subgroups of group $\left[\{1,-1,i,-i\}, \times\right]$. **03**

(b) Define Eulerian graph, Planar Graph. **04**
Justify whether the graph given in Figure 1 is Planer or not using Euler's formula.

(c) Define Binary tree, Spanning tree, Minimal Spanning tree, Find the minimal spanning tree for the weighted graph given in Figure 2. **07**

OR

Q.5 (a) Define Cyclic group, Normal subgroup. Illustrate with an example. **03**

(b) Form a binary search tree for the data 16,24,7,5,8,20,40,3. **04**

(c) Explain Post order traversal. Given the postorder and inorder traversal of a binary tree, draw the unique binary tree. **07**
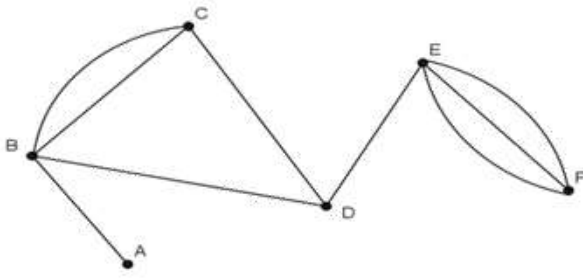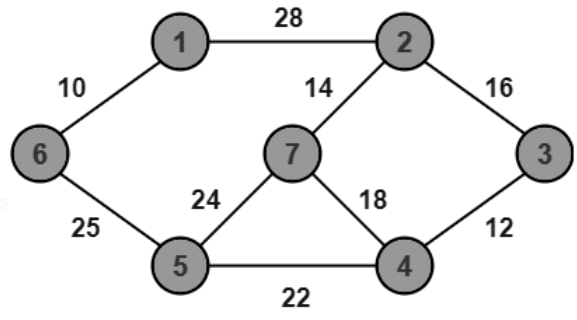Postorder: d e c f b h i g a
Inorder: d c e b f a h g i.

Figure 1

Figure 2

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