

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-I & II(NEW) EXAMINATION – WINTER 2022

Subject Code:3110015

Date:14-03-2023

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q-1**
- (a) If $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$, find $\text{curl}\vec{F}$. 3
- (b) Solve $x\frac{dy}{dx} + y + 1 = 0$ 4
- (c) Evaluate $\int_C [(2xy - x^2)dx + (x + y^2)dy]$ using Green's theorem where C is closed curve formed by $y = x^2$ and $x = y^2$. 7

- Q-2**
- (a) Find the Laplace transform of $t^2 - e^{-2t} + \cosh^2 3t$. 3
- (b) Find the Fourier cosine integral of $f(x) = \frac{\pi}{2}e^{-x}, x \geq 0$. 4
- (c) Using convolution theorem. Find the inverse Laplace transform of $\frac{1}{(s^2+4)^2}$. 7

OR

- (c) If $\vec{F} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$ then 7
- (i) If \vec{F} is conservative, find its scalar potential ϕ .
- (ii) Find the work done in moving a particle under this force field from $(0,1,1)$ to $(1,2,0)$.

- Q-3**
- (a) Find the constant b such that $\vec{A} = (bx + 4y^2z)\hat{i} + (x^3\sin z - 3y)\hat{j} - (e^x + 4\cos x^2y)\hat{k}$ is solenoidal. 3
- (b) Evaluate $\int_0^\infty te^{-2t}\cos t dt$. 4
- (c) By Laplace transform solve, $\frac{dy}{dt} + y = \cos 2t, y(0) = 1$. 7

OR

- Q-3**
- (a) State Second Shifting theorem. Hence find the inverse Laplace transform of $\frac{e^{-as}}{s}$. 3
- (b) Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$. 4
- (c) (i) Solve $y = 2px - xp^2$ 7
(ii) Solve $p = \log(px - y)$

- Q-4** (a) If $y_1 = e^{-x}$ is one of the solutions of $y'' + 2y' + y = 0$, then find the second solution. **3**
- (b) Solve $(D^2 - 2D + 1)y = 10e^x$. **4**
- (c) Find the solution of differential equation $y'' + 4y = 2\sin 3x$ by the method of Undetermined Coefficients. **7**

OR

- Q-4** (a) Solve $2D^2y + Dy - 6y = 0$. **3**
- (b) Solve $(x^2D^2 - 7xD + 12)y = x^2$. **4**
- (c) Using the method of variation of parameter, solve $\frac{d^2y}{dx^2} + y = \sin x$. **7**

- Q-5** (a) Find the ordinary points of $(x^2 + 1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 6y = 0$ **3**
- (b) Solve $(D^2 + 1)y = \sin^2 x$. **4**
- (c) Find power series solution of $\frac{d^2y}{dx^2} + xy = 0$. **7**

OR

- Q-5** (a) Find the value of $J_{\frac{3}{2}}(x)$. **3**
- (b) Find the Laplace transform of $f(t) = \frac{2}{3}t, 0 \leq t \leq 3$, if $f(t) = f(t + 3)$. **4**
- (c) Solve in series the differential equation $4x\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} + y = 0$. **7**