Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110015   **Date: 01/01/2020**
Subject Name: Mathematics –2
Time: 10:30 AM TO 01:30 PM   **Total Marks: 70**
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

		Marks

Q.1 **(a)** Find the length of curve of the portion of the circular helix   **03**
$\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ from $t = 0$ to $t = \pi$

(b) $\int\limits_{(1,2)}^{(3,4)} (xy^2 + y^3)dx + (x^2y + 3xy^2)dy$ is independent of path joining the points   **04**
(1, 2) and (3,4). Hence, evaluate the integral.

(c) Verify tangential form of Green's theorem for $\vec{F} = (x - \sin y)\hat{i} + (\cos y)\hat{j}$,   **07**
where C is the boundary of the region bounded by the lines $y = 0, x = \pi/2$
and $y = x$.

Q.2 **(a)** Find the Laplace transform of $f(t)$ defined as   **03**

$$f(t) = \frac{t}{k} \qquad 0 < t < k$$
$$= 1 \qquad t > k$$

(b) Find the inverse Laplace transform of $\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}$ **04**

(c) (i) Calculate the curl of the vector $xyz\,\hat{i} + 3x^2y\,\hat{j} + (xz^2 - y^2z)\hat{k}$ **07**
(ii) The temperature at any point in space is given by $T = xy + yz + zx$.
Determine the derivative of T in the direction of the vector $3\hat{i} - 4\hat{k}$ at the
point (1, 1, 1).

OR

(c) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$, and $\bar{a}$ is a constant vector. Find the value of   **07**
$$div\left(\frac{\bar{a} \times \bar{r}}{r^n}\right)$$

Q.3 **(a)** Find constants a, b and c such that   **03**
$\vec{V} = (x + 2y + a\,z)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

(b) Using Fourier cosine integral representation show that   **04**
$$\int\limits_0^\infty \frac{\cos \omega x}{k^2 + \omega^2}\,d\omega = \frac{\pi e^{-kx}}{2k}$$

(c) Solve the following differential equations: **07**
    (i) $\cos(x + y)\,dy = dx$

    (ii) $\sec^2 y\,\dfrac{dy}{dx} + x \tan y = x^3$

OR

Q.3 **(a)** Find the Laplace transform of (i) $\int_0^t \dfrac{\sin t}{t} dt$ (ii) $t^2 u(t-3)$ **03**

(b) Using Convolution theorem obtain $L^{-1}\left(\dfrac{1}{s(s^2+a^2)}\right)$ **04**

(c) Find the power series solution of $\dfrac{d^2 y}{dx^2} + xy = 0$ **07**

Q.4 **(a)** Find the Laplace transform of the waveform   **03**

$f(t) = \left(\dfrac{2t}{3}\right), 0 \le t \le 3$

(b) Using the Laplace transforms, find the solution of the initial value problem   **04**
$y'' + 25y = 10\cos 5t \quad y(0) = 2, y'(0) = 0$

(c) Using variation of parameter method solve $(D^2 +1)y = x\sin x$ **07**

OR

Q.4 **(a)** Solve $y\dfrac{d^2 y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = \dfrac{dy}{dx}$ **03**

(b) Solve $y''' - 3y'' + 3y' - y = 4e^t$ **04**

(c) Solve $\dfrac{d^2 y}{dx^2} + 2\dfrac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ using method of undetermined coefficients.   **07**

Q.5 **(a)** Classify the singular points of the equation $x^3(x-2)y'' + x^3 y' + 6y = 0$ **03**

(b) Solve $(D^2 +4)y = \cos 2x$ **04**

(c) Solve (i) $ye^x dx + (2y + e^x)dy = 0$ (ii) $\dfrac{dy}{dx} + 2y\tan x = \sin x$ **07**

OR

Q.5 **(a)** Solve $\dfrac{dy}{dx} = \dfrac{y^3}{e^{2x} + y^2}$ **03**

(b) If $y_1 = x$ is one of solution of $x^2 y'' + xy' - y = 0$ find the second solution.   **04**

(c) Using Frobenius method solve $x^2 y'' + 4xy' + (x^2 + 2)y = 0$ **07**

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