

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- I & II (NEW) EXAMINATION – WINTER 2019****Subject Code: 3110014****Date: 17/01/2020****Subject Name: Mathematics – I****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		MARKS
Q.1	(a) Find the equations of the tangent plane and normal line to the surface $x^2 + y^2 + z^2 = 3$ at the point (1,1,1)	03
	(b) Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$	04
	(c) Using Gauss Elimination method solve the following system $-x+3y+4z=30$ $3x+2y-z=9$ $2x-y+2z=10$	07
Q.2	(a) Test the convergence of the series $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \dots + \left(\frac{n}{2n+1}\right)^n + \dots$	03
	(b) Discuss the Maxima and Minima of the function $3x^2 - y^2 + x^3$	04
	(c) Find the fourier series of $f(x) = \frac{(\pi-x)}{2}$ in the interval (0,2 π)	07
OR		
(c)	Change the order of integration and evaluate $\int_0^1 \int_x^1 \sin y^2 dy dx$	07
Q.3	(a) Find the value of $\beta \left(\frac{7}{2}, \frac{5}{2}\right)$	03
	(b) Obtain the fourier cosine series of the function $f(x) = e^x$ in the range (0,l)	04
	(c) Find the maximum and minimum distance from the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 36$	07
OR		
Q.3	(a) Test the convergence of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$	03
	(b) Evaluate $\iint (x^2 - y^2) dx dy$ over the triangle with the vertices (0,1), (1,1), (1,2)	04
	(c) Find the volume of the solid generated by rotating the plane region bounded by $y = \frac{1}{x}$, $x=1$ and $x=3$ about the X axis.	07
Q.4	(a) Evaluate $\int_0^\pi \int_0^{\sin \theta} r dr d\theta$	03
	(b) Express $f(x) = 2x^3 + 3x^2 - 8x + 7$ in terms of (x-2)	04

- (c) Using Gauss-Jordan method find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **07**

OR

- Q.4** (a) Using Cayley-Hamilton Theorem find A^{-1} for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **03**

- (b) Evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$ **04**

- (c) Test the convergence of the series **07**
 $\frac{x}{1 \cdot 2} + \frac{x^2}{3 \cdot 4} + \frac{x^3}{5 \cdot 6} + \frac{x^4}{7 \cdot 8} + \dots$

- Q.5** (a) Evaluate $\int_0^1 \int_1^2 xy \, dy \, dx$ **03**

- (b) Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$ **04**

- (c) If $u = f(x-y, y-z, z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **07**

OR

- Q.5** (a) Find the directional derivatives of $f = xy^2 + yz^2$ at the point $(2, -1, 1)$, in the direction of $i+2j+2k$. **03**

- (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ **04**

- (c) Evaluate $\iiint xyz \, dx \, dy \, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = 4$ **07**
