Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (NEW SYLLABUS) – WINTER 2018

Subject Code: 2110015 **Date: 03/01/2019**

Subject Name: Vector Calculus and Linear Algebra

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Marks**

(a) **07**

1. For a square matrix A, if $A \cdot A^T = I$, then A is
(a) Symmetric (b) Skew-symmetric (c) Orthogonal (d) Singular

2. For inner product defined by $\langle p, q \rangle = \int_{-1}^{1} p \cdot q \, dx$, value of $\|x\|$ is
(a) $\sqrt{2/3}$ (b) $2/3$ (c) 0 (d) 2

3. The number of vectors in a basis for vector space is known as
(a)degree (b) order (c) dimension (d) zeros

4. Which is the standard basis vector of R^3?
(a)$(1,1,1)$ (b) $(1,0,1)$ (c) (0,0,0) (d) $(1,0,0)$

5. Which of the following is in reduced row echelon form?
(a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6. For homogeneous system of equations $AX = 0$, if $|A| \neq 0$, then the system has
(a)trivial solution (b) non trivial solution (c) infinite no. of solutions
(d) no solution

7. Which is not correct for the matrices?
(a)$(A^T)^T = A$ (b) $(AB)^T = A^T B^T$ (c) $A^0 = I$ (d) None of these

(b) **07**

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is
(a) negative definite (b) positive definite
(c) indefinite (d) positive semi definite

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 5 & 0 \\ -7 & 8 & 2 \end{bmatrix}$, then eigen values of A^{-1} are
(a) $1,5,2$ (b) $-1,-5,-2$ (c) $0,0,0$ (d) $1, \frac{1}{5}, \frac{1}{2}$

3. Which of the following linear transformation is one to one?
(a)$T(x,y) = (8x + 4y, 2x + y)$ (b) $T(x,y) = (x + 2y, -x + y)$
(c) $T(x,y) = (4x - 6y, -2x + 3y)$ (d) None of these

4. Find the value of 'a' if $\bar{F} = (x + 3y)i + (y - 2z)j + (az + x)k$ is solenoidal.
(a)2 (b)1 (c)-2 (d)0

5. Rank of the identity matrix I_n is
(a) n (b) n-1 (c) n+1 (d) 1

6. For which value of k, (2,1,3) and (1,7, k) are orthogonal?
(a) -3 (b) -1 (c) 0 (d) 2

7. Dimension of $P_2 = \{a_0 + a_1 x + a_2 x^2 / \ a_i \in R, \ i = 0,1,2\}$ is
(a) 0 (b) 1 (c) 2 (d) 3

Q.2 (a) Prove by Wronskian that $1, x, x^2$ are Linearly independent. **03**

(b) Is $\{(2,1),(3,0)\}$ a basis for R^2? **04**

(c) Check whether the set of all $(x,y) \in R^2$ with binary operations $(x,y)+(x',y') = (x+x', y+y')$ and $k(x,y) = (kx, y)$ is vector space or not. **07**

Q.3 (a) Determine whether $T: R^2 \to R^2$ is a linear transformation. **03**
$T(x,y) = (x+1, y)$

(b) Solve by Cramer's rule. **04**
$x_1 + 2x_3 = 6$
$-3x_1 + 4x_2 + 6x_3 = 30$
$-x_1 - 2x_2 + 3x_3 = 8$

(c) Find a standard basis vector that can be added to the set $\{v_1, v_2\}$ to produce a basis for R^3. **07**
$v_1 = (-1, 2, 3), \quad v_2 = (1, -2, -2)$

Q.4 (a) Find values of a, b for which A and B are not invertible matrices. **03**
$A = \begin{bmatrix} a+b-1 & 0 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 \\ 0 & 2a-3b-7 \end{bmatrix}$

(b) Find rank of the matrix by the determinant method. **04**
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -2 & -3 & -4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(c) Solve by Gauss-Jordan elimination. **07**
$2x_1 + 2x_2 + 2x_3 = 0$
$-2x_1 + 5x_2 + 2x_3 = 1$
$8x_1 + x_2 + 4x_3 = -1$

Q.5 (a) Let $u = (u_1, u_2), v = (v_1, v_2) \in R^2$. **03**
Verify that $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$ is an inner product.

(b) Find the directional derivative of $xy + yz + zx$ in the direction of $3i - 2j + k$ at $(1,2,3)$. **04**

(c) Use Gram-Schmidt process to transform basis $\{u_1, u_2, u_3\}$ into an orthonormal basis with Euclidean inner product. **07**
$u_1 = (1,1,1), u_2 = (-1,1,0), u_3 = (1,2,1)$

Q.6 (a) Verify Caley – Hamilton theorem for $A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$. **03**

(b) For linear transformation **04**
$T: R^2 \to R^2, T(x,y) = (2x - y, -8x + 4y)$, which of the following are in $Ker(T)$?
(i) $(5,10)$ (ii) $(3,2)$

(c) Identify the curve. **07**
$9x^2 + 4y^2 - 36x - 24y + 36 = 0$

Q.7 (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$. **03**

(b) Find the work done in moving a particle in the force field $\bar{F} = 3x^2 i + (2xz - y) j + zk$ along the straight line from $(0,0,0)$ to $(2,1,3)$. **04**

(c) Using Green's theorem evaluate $\int_c (x^2y\, dx + x^2\, dy)$ where c is the boundary of the triangle whose vertices are $(0,0), (1,0), (1,1)$. **07**

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