Seat No./Enrollment No.: ________

GUJARAT TECHNOLOGICAL UNIVERSITY

BACHELOR OF ENGINEERING – SEMESTER 3 – EXAMINATION – SUMMER 2026

Subject Code: 3130107 **Date: 03/07/2026**

Subject Name: Partial Differential Equations and Numerical Methods

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 **(a)** Find the real root of the equation $f(x) = x^3 - x - 1 = 0$ correct upto two decimal places. **03**

(b) Use Bisection method to find a real root of $2x - \log_{10} x = 7$ correct upto five decimal places. **04**

(c) Explain Newton-Raphson method briefly and use it to obtain approximate root of the equation $f(x) = x^4 - x - 9 = 0$ correct to three places of decimal. **07**

Q.2 **(a)** Use Trapezoidal rule to evaluate **03**

$$\int_4^8 \frac{dx}{x}$$

using four sub-intervals.

(b) Solve the following linear system of equations using Gauss elimination method. **04**

$$3x + y - z = 3$$
$$2x - 8y + z = -5$$
$$x - 2y + 9z = 8$$

(c) Compute $\cosh(0.56)$ using Newton's forward difference formula and also estimate the error for the following table. **07**

x	0.5	0.6	0.7	0.8
$f(x)$	1.127626	1.185465	1.255169	1.337435

OR

Q.2 **(c)** Evaluate $\int_0^1 e^{-x^2} dx$ using **07**

1. Trapezoidal rule with $n = 10$ sub-intervals

Simpson's-$^1/_3$ rule with n=10 sub-intervals.

Q.3 **(a)** Evaluate **03**

$$\int_0^1 \frac{dx}{1 + x^2}$$

using Simpson's-$^3/_8$ rule taking $h = \frac{1}{6}$.

(b) Find approximate solution of following linear system of equations using Gauss-seidel iteration method. **04**

$$2x + y + z = 4$$
$$x + 2y + z = 4$$
$$x + y + 2z = 4$$

(c) Explain Euler's method briefly and use it to evaluate $y(1.0)$ and $h = 0.2$ to solve the following initial value problem. **07**

$\frac{dy}{dx} = x + y | y(0) = 0$

Also compare the solution with the exact solution of the problem.

OR

Q.3 (a) Use Runge-kutta fourth order method to evaluate $y(0.3)$ for following Initial value problem **03**

$\frac{dy}{dx} = 1 - \frac{y^2}{4} | y(0) = 1$

Taking $h = 0.1$ correct to five decimals of places.

(b) Distance in nautical miles of the visible horizon for given heights in meters above the surface of the earth given by the following table. **04**

x (Heights)	100	150	200	250	300	350	400
y (Distance)	12	16	21	27	36	50	72

Find the value of y when $x = 225$ meters.

(c) Using Taylor's series method solve $\frac{dy}{dx} = x^2 - y | y(0) = 1$ at $x = 0.1, 0.2, 0.3$ and 0.4. Compare with the exact solution. **07**

Q.4 (a) Fit a straight line to the following data. **03**

x	71	68	73	69	67	65	66	67
y	69	72	70	70	68	67	68	64

(b) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$ **04**

(c) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - 1 = \cos(x + 2y)$ **07**

OR

Q.4 (a) Fit the curve $y = ae^{bx}$ for the following data. **03**

x	1	5	7	9	12
y	10	15	12	15	21

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0$ **04**

(c) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + 3\frac{\partial z}{\partial y} - \frac{\partial z}{\partial x} = e^{(x+2y)} + xy$ **07**

Q.5 (a) Solve $\frac{y^2 z}{x} p + xzq = y^2$ **03**

(b) Solve $\sqrt{p} + \sqrt{q} = 1$ **04**

(c) Obtain the solution of following one-dimensional heat equation with insulated sides by the method of separation of variables. **07**

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

$u(0,t) = u(l,t) = 0 \ \forall t > 0$

$u(x,0) = f(x)$ for $0 < x < l$

OR

Q.5 (a) Solve $x^2(y - z)p + y^2(z - x) = z^2(x - y)$ **03**

(b) Solve $z^2(p^2 + q^2 + 1) = a^2$ **04**

(c) Obtain the solution of following one-dimensional heat equation with insulated ends by the method of separation of variables. **07**

$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

$u_x(0,t) = u_x(l,t) = 0 \ \forall t > 0$

$u(x,0) = f(x)$ for $0 < x < l$

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