Seat No./Enrollment No.: _______

GUJARAT TECHNOLOGICAL UNIVERSITY
BACHELOR OF ENGINEERING – SEMESTER 3 – EXAMINATION – SUMMER 2026

Subject Code: 3130005 **Date: 03/07/2026**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1	**(a)**	Express the complex number $(1+\sqrt{3}\,i)^2$ in polar form.	**03**
	(b)	Solve the partial differential equation: $(D^2 - DD' - 12D'^2)z = e^{2x-3y}$.	**04**
	(c)	Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate $v(x,y)$.	**07**
Q.2	**(a)**	Discuss about the types of singularities of f(z).	**03**
	(b)	Evaluate $\int_C (x^2 + ixy)dz$; from A (1,1) to B (2,4) along the curve $y = x^2$.	**04**
	(c)	Find the bilinear transformation mapping $z_1 = 1, z_2 = 0, z_3 = -1$ to $w_1 = i, w_2 = \infty, w_3 = 1$.	**07**
		OR	
Q.2	**(c)**	Determine an analytic function with real part $u(x,y) = \frac{y}{x^2 + y^2}$.	**07**
Q.3	**(a)**	Find all values of $(-8i)^{\frac{1}{3}}$ using De' Moivre's theorem.	**03**
	(b)	Evaluate $\int_C \frac{cos\pi z^2}{(z-1)(z-2)}dz$, where C is $\lvert z\rvert = 5/2$.	**04**
	(c)	Find the Laurent series of $\frac{1}{z(z-1)(z-2)}$ for the regions: (i) 0 < |z| < 1, (ii) 1 < |z| < 2, (iii) |z| > 2.	**07**
		OR	
Q.3	**(a)**	State Cauchy- Reimann Equations and show that the function $f(z) = xy + iy$ is nowhere analytic.	**03**
	(b)	Find Maclaurin series expansion of $f(z) = \frac{1}{z+3i}$ and its radius of convergence.	**04**
	(c)	Find the residues at singular points of $f(z) = \frac{z^2}{(z-1)(z+2)}$. Using Cauchy's Residue theorem evaluate $\int_C f(z)dz$, where C is |z+1| =1.	**07**
Q.4	**(a)**	Form partial differential equation from the equation $z = xy + f(x^2 + y^2)$ by eliminating arbitrary function.	**03**
	(b)	Classify the second order homogeneous partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 12\frac{\partial^2 z}{\partial y^2} = 0$ as elliptic, parabolic or hyperbolic.	**04**
	(c)	(i) Solve the linear partial differential equation $xp + yq = 3z$. (ii) Find the complete integral of $p^2 = q + x$.	**07**

Q.4 **(a)** Form partial differential equation from OR $z = ax + by + a$ by eliminating the arbitrary constant a and b. **03**

(b) Solve the second order homogeneous partial differential equation: $z_{xx} - 2z_{xy} - 8z_{yy} = 0$. **04**

(c) Using the Charpit's method solve the non linear partial differential equation: $px + qy = pq$. **07**

Q.5 **(a)** Solve the partial differential equation using the method of separation of variables: $u_x + 2u_y = 0$, given that $u(x, 0) = 4e^{-x}$. **03**

(b) Evaluate $\int_C \frac{e^{-z}}{z+1} dz$, where C is (i) $|z-1|=1$, (ii) $|z+1|=1$ **04**

(c) A tightly stretched string with fixed ends x = 0 and x = L is initially in a position given by $u(x, 0) = u_0 sin^3(\pi x/L)$. If it released at rest from this position, find the displacement $u(x, t)$. **07**

OR

Q.5 **(a)** Using the method of separation of variables, solve $u_x = 2u_t + u$ with $u(x, 0) = 6e^{-3x}$. **03**

(b) Find Upper bound of absolute value of the integral $\oint_C e^z dz$, where C is a line segment joining point (0,0) and $(1, 2\sqrt{2})$. **04**

(c) Solve Laplace's equation in square lamina with $u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and $u(x, 0) = T_0$, constant. **07**

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