

GUJARAT TECHNOLOGICAL UNIVERSITY**BACHELOR OF ENGINEERING – SEMESTER 4 – EXAMINATION – SUMMER 2026**

Subject Code: 2141905

Date: 19/05/2026

Subject Name: Complex Variables and Numerical Methods

Time: 10:30 AM TO 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1**
- (a) Find modulus and principal argument of $z = \frac{1-7i}{(2+i)^2}$. 03
- (b) If $f(z) = \begin{cases} \frac{x^3-y^3}{x^3+y^3}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$ Show that $f(z)$ is not continuous at the origin. 04
- (c) Solve the given system of linear equations by Gauss-elimination method. 07
 $x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40.$
- Q.2**
- (a) Sketch $S = \{z / -1 \leq I_m(z) \leq 2\}$. Is it connected? 03
- (b) Find and plot all roots of $\sqrt[3]{81}$. 04
- (c) Show that $u = x^3 - 3x^2y$ is a harmonic function. Also find its harmonic conjugate. 07
- OR
- (c) Show that $\tan\left(i \log\left(\frac{x+iy}{x-iy}\right)\right) = \frac{2xy}{y^2-x^2}$. 07
- Q.3**
- (a) Determine and sketch the region where vertices are $2, 1+i, 1-i$, under the transformation $w = z + 3 - 4i$. 03
- (b) Evaluate $\int_C (x^2 - iy^2) dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$. 04
- (c) Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the region 07
 $(i) |z| < 2, (ii) 2 < |z| < 4, (iii) |z| > 4$
- OR
- Q.3**
- (a) Classify the singularities of $f(z) = \frac{\sinh z}{z}$. 03
- (b) Discuss the convergence of $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z-3i)^n$ and also find the radius of convergence. 04
- (c) Evaluate $\int_C \frac{dz}{(z^2+1)^2}$ where $C: |z+i| = 1$. 07
- Q.4**
- (a) Prove that: $\Delta = E - 1, \nabla = 1 - E^{-1}$. 03
- (b) By Newton-Raphson method find the positive root of $x = \cos x$ upto 3 decimal places. 04

- (c) Apply Gauss-elimination method to solve the following system of linear equations: 07

$$2x + y - z = 1, 5x + 2y + 2z = -4, 3x + y + z = 5$$

OR

- Q.4 (a) State Trapezoidal rule with $n = 10$ and evaluate $\int_0^1 e^x dx$. 03

- (b) From the following data, find the number of person earning weekly wages between 100 and 110 rupees. 04

Wages inRs.	Below 80	80-100	100-120	120-140	140-160
No. of Person(in thousand)	250	120	100	70	50

- (c) 07

Find the largest eigen value for $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- Q.5 (a) Write Gaussian formula for one, two and three points. 03

- (b) Using Taylor's Series method, to solve $\frac{dy}{dx} = x^2y - 1, y(0) = 1$. Also find $y(0.03)$. 04

- (c) Apply Runge-Kutta method of fourth order to calculate $y(0.2)$, given that 07

$$\frac{dy}{dx} = x + y, y(0) = 1, \text{ and } h = 0.1$$

OR

- Q.5 (a) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^6 \frac{dx}{1+x}$ by taking $h = 1$. 03

- (b) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table given below: 04

x	-1	0	1	3
y	2	1	0	-1

- (c) Solve the following equations by Gauss-Seidel method correct up to two decimal places. 07

$$10x + y + z = 6, x + 10y + z = 6, x + y + 10z = 6.$$
