Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II (NEW) EXAMINATION – SUMMER 2024

Subject Code:3110015 **Date:02-07-2024**

Subject Name:Mathematics - 2

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Find curl of $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at (2, -1, 1). **03**

(b) If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy – plane from (0,0) to (1,4) along a curve $y = 4x^2$. Find the work done. **04**

(c) State and apply Green's theorem to evaluate $\int_C \left[(2x^2 - y^2)dx + (x^2 + y^2)dy\right]$, where C is the boundary of the area enclosed by the x – axis and the upper half of the circle $x^2 + y^2 = a^2$. **07**

Q.2 **(a)** Find Laplace transform of $f(t) = \int_0^t \frac{\sin t}{t} dt$. **03**

(b) Find the Fourier cosine integral of $f(x) = e^{-kx}$, where $x > 0, k > 0$ **04**

(c) State convolution theorem and use it to find inverse Laplace transform of $\frac{1}{(s^2 + a^2)^2}$. **07**

OR

(c) Using Laplace transform solve the following initial value problem $y'' + 4y' + 8y = 1,\ y(0) = 0, y'(0) = 1.$ **07**

Q.3 **(a)** Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$. **03**

(b) Find the inverse Laplace transform of $\frac{se^{-\frac{s}{2}} + \pi e^{-s}}{s^2 + \pi^2}$. **04**

(c) Solve (i) $y + px = x^4p^2$ **07**

(ii) $p^2 - xp + y = 0$

OR

Q.3 **(a)** Solve $(y^2 - x^2)dx + 2xy\,dy = 0$ **03**

(b) Find the Laplace transform of the waveform **04**

$f(t) = \left(\frac{2t}{3}\right), 0 \le t \le 3.$

	(c)	Find the series solution of $\left(1+x^2\right)y''+xy'-9y=0$.	**07**
Q.4	**(a)**	Solve $9yy'+4x=0$.	**03**
	(b)	If $y_1=x$ is one of the solution of $x^2y''+xy'-y=0$,find the second solution.	**04**
	(c)	Using the method of variation of parameter, solve $\dfrac{d^2y}{dx^2}+y=\sin x$.	**07**
		OR	
Q.4	**(a)**	Find Laplace transform of $t^2u\left(t-2\right)$.	**03**
	(b)	Solve $\left(D^2+9\right)y=2\sin 3x+\cos 3x$. where $D=\dfrac{d}{dx}$	**04**
	(c)	Using the method of undetermined coefficients, solve $y''-2y'+5y=5x^3-6x^2+6x$.	**07**
Q.5	**(a)**	Solve $x^2y''-20y=0$.	**03**
	(b)	Solve $\left(D^2-1\right)y=xe^x$ where $D=\dfrac{d}{dx}$	**04**
	(c)	Using Frobenius method, solve $4x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=0$.	**07**
		OR	
Q.5	**(a)**	Classify the singular points of the equation $x^3\left(x-2\right)y''+x^3y'+6y=0$.	**03**
	(b)	Prove that $\dfrac{d}{dx}\left[J^2{}_n(x)\right]=\dfrac{x}{2n}\left[J^2{}_{n-1}(x)-J^2{}_{n+1}(x)\right]$.	**04**
	(c)	Show that $\int_{-1}^{1}x^2P_{n-1}(x)P_{n+1}(x)\,dx=\dfrac{2n\left(n+1\right)}{\left(2n-1\right)\left(2n+1\right)\left(2n+3\right)}$.	**07**

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