Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II (NEW) EXAMINATION – SUMMER 2024

Subject Code:3110014 **Date:09-07-2024**

Subject Name:Mathematics - 1

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Using L' Hospital's rule, evaluate $\lim_{x\to 1}\frac{x-x^x}{1+\log x-x}$.	**03**
	(b)	Define Beta function and evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$.	**04**
	(c)	Find the Fourier series of $f(x)=\begin{cases}\pi+x & -\pi<x<0\\ \pi-x & 0<x<\pi\end{cases}$	**07**
Q.2	**(a)**	Show that the sequence $\{u_n\}$, where $u_n=\frac{\sin n}{n}$ converges to zero.	**03**
	(b)	Express $f(x)=2x^3+3x^2-8x+7$ in terms of $(x-2)$.	**04**
	(c)	Find the area of the surface of revolution of a quadrant of a circular arc as obtained by revolving it about a tangent at one of its ends.	**07**
		OR	
	(c)	Find the length of the loop of the curve $9ay^2=(x-2a)(x-5a)^2$.	**07**
Q.3	**(a)**	Evaluate $\int_0^\infty \frac{dv}{\left(1+v^2\right)\left(1+\tan^{-1}v\right)}$.	**03**
	(b)	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$.	**04**
	(c)	If $\theta=t^n e^{-\frac{r^2}{4t}}$ then find n so that $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$.	**07**
		OR	
Q.3	**(a)**	Check the convergence of $\int_0^5 \frac{1}{x^2}dx$.	**03**
	(b)	Evaluate $\int_0^\infty 3^{-4x^2}dx$.	**04**
	(c)	Find the Fourier series of $f(x)=\frac{1}{2}(\pi-x)$ in the interval $(0,2\pi)$. Hence, deduce that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....$	**07**

Q.4 **(a)** Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$ **03**

(b) Find the Fourier sine series of $f(x) = e^x$ in $0 < x < \pi$. **04**

(c) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **07**

OR

Q.4 **(a)** Find the directional derivative of $f(x, y, z) = x^2 yz + 4xz^2$ at (1, -2, -1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **03**

(b) Solve the following system by Gauss – Jordan method: **04**

$$-2y + 3z = 1$$
$$3x + 6y - 3z = -2$$
$$6x + 6y + 3z = 5$$

(c) Change the order of integration and evaluate **07**

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\, dx\, dy.$$

Q.5 **(a)** Evaluate $\int_0^3 \int_0^1 \left(x^2 + 3y^2\right) dy\, dx.$ **03**

(b) Apply Cayley – Hamilton theorem to $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and deduce that $A^8 = 625I.$ **04**

(c) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}\, dy\, dx$ by changing to polar coordinates. **07**

OR

Q.5 **(a)** Evaluate $\int_0^2 \int_1^2 \int_0^{yz} xyz\, dx\, dy\, dz$. **03**

(b) Using Gauss Jordan method, find inverse of $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. **04**

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. **07**

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