

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023****Subject Code:3144007****Date:19-07-2023****Subject Name:Numerical and Statistical Methods****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.
5. Statistical tables need to be provided.

MARKS

Q.1 (a) Four cards are drawn from a pack of cards. Find the probability that there are two spades and two hearts. **03**

(b) Determine which section is more skewed from the following data: **04**
 Section A: $\bar{x} = 35$, s.d = 14.3, mode = 37.12
 Section B: $\bar{x} = 32$, s.d = 14.3, mode = 27.8

(c) Using Newton's interpolation formula calculate P when $t = 142^{\circ}\text{C}$ from the following data: **07**

Temp $t^{\circ}\text{C}$	140	150	160	170	180
Pressure P	3.685	4.854	6.302	8.076	10.225

Q.2 (a) Given that $P(A) = 3/8$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$. Find $P(A|B)$. Are A and B independent? **03**

(b) Evaluate $\int_0^1 \sqrt{1+x^3} dx$ taking $h = 0.2$, correct to three decimal places by Trapezoidal rule. **04**

(c) From the following table, obtain the two regression equations and estimate (i) the yield of crops (Y) when the rainfall (X) is 29 and (ii) rainfall (X) when yield (Y) is 600. Correlation coefficient $r_{xy} = 0.52$ **07**

	Yield (Y)	Rainfall (X)
Mean	508.4	23.7
Standard Deviation	36.8	4.6

OR

(c) It is estimated that 50% of emails are spam. A certain brand of software claims that it can detect 99% of spam emails, and the probability that a non-spam email detected as spam is 5%. If an email is detected as spam, then what is the probability that it is in fact a non-spam email? **07**

Q.3 (a) Use Secant method to find the approximate root of the equation $x^2 - 2 = 0$ lying in the interval (1,1.5) correct up to two decimal places. **03**

(b) Calculate the rank correlation coefficient from the following data: **04**

Rank in Statistics	1	2	3	4	5	6	7	8
Rank in Mathematics	2	4	1	5	3	8	7	6

- (c) Let X be a continuous random variable with density function 07

$$f(x) = \begin{cases} cx^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) the constant c and (ii) $E(X)$ and $\text{Var}(X)$.

OR

- Q.3 (a)** The random variable X is distributed in a Poisson form. If 03
 $P(X = 1) = P(X = 2)$, then evaluate $P(X = 0)$.

- (b) Scores of a competitive examination are normally distributed with a mean of 527 and standard deviation of 112. What is the probability of an individual scoring above 500? 04

- (c) Consider the ordinary differential equation 07

$$\frac{dy}{dx} = x^2 + y^2, \quad y(1) = 1.2$$

Find $y(1.05)$ using the fourth order Runge-Kutta method.

- Q.4 (a)** On a certain examination, the average grade of all students in class A is 68.4 and all students in class B is 71.2. If the average of both classes combined is 70, what is the ratio of the number of students in class A to the number in class B? 03

- (b) A random sample of 10 individuals is drawn from a population having sample mean 27 and sample variance 20. Assuming that the population is approximately normally distributed, can we conclude that the population mean is different from 30 years? Test at 5% significant level. 04

- (c) Fit a straight line to the data given below and estimate the value of y when $x = 2.5$ 07

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

OR

- Q.4 (a)** The correlation coefficient between income and food expenditure for sample of 7 household from a low-income group is 0.9. Using 1% level of significance, test whether the correlation coefficient is positive. Assume that the population is normally distributed. 03

- (b) A machine produced 20 defective articles in a batch of 400. After repairing, it produced 10 defectives in a batch of 300. Has the machine improved? Test at 5% significant level. 04

- (c) Fit the curve $y = ax^b$ to the following data: 07

x	2	3	4	5
y	27.8	62.1	110	161

- Q.5 (a)** The random variable X follows the probability distribution: 03

X	0.2	0.4	0.5	0.8	1
p(X = x)	0.1	0.2	0.2	0.3	0.2

Evaluate (i) $P(X = 0.2 | X < 0.6)$.

- (b) The variances calculated from two random samples of sizes 16 and 21 are 2.09 and 1.10. May the samples be regarded as drawn from normal populations with the same standard deviation? (Test at 5% significant level) 04

- (c) Use Lagrange's interpolation formula to calculate $f(301)$ from the following data: **07**

x	300	304	305	307
f(x)	2.4771	2.4829	2.4843	2.4871

OR

- Q.5 (a) Using Euler's method, find $y(0.5)$ where $\frac{dy}{dx} = x + 2y$, $y(0) = 0$ taking $h = 0.25$. **03**

- (b) Sugar is packed in bags by an automatic machine with mean content of bags as 1 kg. A random sample of 36 bags is selected and mean mass has been found to be 1.003 kg. If a standard deviation of 0.01 kg is acceptable on all the bags being packed, determine on the basis of sample test at 5% significant level, whether the machine requires adjustment. **04**

- (c) Use Chi-Square distribution to test the hypothesis at 5% level of significance that gender of a baby is related to the baby's heart rate from the following data of 40 records: **07**

Gender	Heart Rate	
	Low	High
Girl	11	7
Boy	17	5
