Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW) EXAMINATION – SUMMER 2023

Subject Code:3110015 Date:07-08-2023

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find the directional derivative of $f(x,y,z) = x^2 + 5y^2 + 3z^2$ at point (1,1,1) in the direction 3i+4j+5k.	**03**
	(b)	A vector field is given by $F = 3x^2yz\,i + x^3z\,j + x^3y\,k$, show that f is irrotational and find the scalar $\emptyset$ such that $F = \nabla\emptyset$.	**04**
	(c)	Verify the Greens theorem for $F = x^2i + xy^2\,j$, along the square bounded by $x = 0, \quad x = 1, \quad y = 0\ and\ y = 1$	**07**
Q.2	**(a)**	Discuss about ordinary point, singular point, regular singular point and irregular singular point of differential equation $(x^2-1)y'' + 3xy + 5x^2y = 0$	**03**
	(b)	Express $f(x) = x^3 + 2x^2 + 3x - 1$ in terms of Legendre's polynomial.	**04**
	(c)	Find a power series solution of $y'' - 25y = 0$ near an ordinary point x=0.	**07**
		OR	
	(c)	Prove that $\mathrm{Pn(x)} = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$	**07**
Q.3	**(a)**	Find the Laplace transform of (i) $e^{3t}t^5$ (ii) t cos3t (iii)($\sin^2 5t)/t$	**03**
	(b)	Find the Inverse Laplace transform of (i) $\frac{1}{(s-1)(s-2)(s-3)}$ (ii) $\frac{1}{s^4-9s^2}$ (iii) $\frac{s+1}{s^2+2s+10}$ (iv) $tan^{-1}(\frac{s}{4})$	**04**
	(c)	Solve the initial value problem using Laplace transformation $y'' - 5y' - 6y = e^{3t}$ with $y(0) = 3\ and\ y'(0) = 2$	**07**
		OR	
Q.3	**(a)**	Find the Laplace transform of (i) $e^{4t}u(t-2)$ (ii) $(t^2+1)u(t-1)$ (iii) $sin3t\,u(t-\pi)$	**03**
	(b)	Using convolution theorem find the Inverse Laplace transform of $\frac{1}{(s^2-4)(s^2-9)}$	**04**
	(c)	Find the Fourier integral of	**07**

$$f(x) = \begin{cases} 0, & |x| < 5 \\ 5, & |x| > 5 \end{cases}$$

Q.4 **(a)** Solve the Differential equation **03**

$e^x cosy\, dx - e^x siny\, dy = 0$

(b) Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0,\ y(0) = 1,\ y'(0) = 2$ **04**

(c) Solve **07**

(i) $\frac{dy}{dx} + y\, tanx = sin2x,\quad y(0) = 2$

(ii) $(x^3 + y^3)dx - xy^2dy = 0$

OR

Q.4 **(a)** Solve $(x^2 - y^2)dx + xydy = 0$ **03**

(b) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 14y = 0,\ y(0) = 3,\ y'(0) = 1$ **04**

(c) Solve **07**

(i) $(y - px)(p^2 + 1) = tan^{-1}p$

(ii) $p^2x^2 = x^2 + p^2$

Q.5 **(a)** Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = e^{-4x} - 12x$ **03**

(b) Using method of undetermined coefficient obtain the solution of **04**

$\frac{d^2y}{dx^2} + 4y = sinx$

(c) Find the solution of differential equations **07**

(1) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2e^x$

(ii) $\frac{x^2d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = \cos(logx)$

OR

Q.5 **(a)** Solve **03**

$$\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = cos2x$$

(b) Using method of undetermined coefficient **04**

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x^2$

(c) (i) Using method of variation of parameters find the solution of differential equation $\frac{d^2y}{dx^2} + 16y = cot4x$ **07**

(ii) Find the solution of differential equation

$$\frac{x^2d^2y}{dx^2} - 6x\frac{dy}{dx} + 6y = x^2 + \frac{1}{x^2}$$

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