Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW) EXAMINATION – SUMMER 2023

Subject Code:3110014 **Date:31-07-2023**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Evaluate: $\lim_{x\to 0}\frac{e^x - e^{-x} - 2x}{x - \sin x}$.	**03**
	(b)	Show that series $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is convergent. Hence find sum of the series.	**04**
	(c)	Define eigenvalue and eigen vector of a matrix. Find all eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.	**07**
Q.2	**(a)**	Evaluate: $\int_0^1\int_x^{2x}\int_0^y 2xyz\, dzdydx$.	**03**
	(b)	Find equations of tangent plane and normal line to the surface $\frac{x^2}{2} - \frac{y^2}{3} - z = 0 \; at \; (2,3,-1)$.	**04**
	(c)	Find Fourier series of 2π -periodic function $f(x) = x + x^2; -\pi < x < \pi.$	**07**
		OR	
	(c)	Find the Fourier series for the periodic extension of $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$.	**07**
Q.3	**(a)**	Define Improper integrals of type-I. Evaluate $\int_{-1}^{\infty}\frac{dt}{t^2 + 5t + 6}$.	**03**
	(b)	Find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 0 \; and \; x = 2$ about the $x-$axis.	**04**
	(c)	(i) Using ratio test discuss convergence of series $\sum_{n=1}^{\infty}\frac{(n+3)!}{3!n!3^n}$.	**03**
		(ii) Use Taylor series to represent function $f(x) = 2x^3 + x^2 + 3x - 8$ in powers of $(x-1)$.	**04**
		OR	

Q.3	**(a)**	Define Beta function. Evaluate $\int_0^1 x^4(1-\sqrt{x})^5 dx$.	**03**
	(b)	Find the arc length of $f(x)=\ln(\cos x), \quad 0 \le x \le \pi/4$.	**04**
	(c)	(i) Using Sandwich theorem find limit of sequence $(an)=\left(\dfrac{\cos n}{n}\right)$.	**03**
		(ii) Define Radius of Convergence of power series. Find it for power series $\sum_{n=1}^{\infty}\dfrac{(x-1)^n}{n^3 3^n}$.	**04**
Q.4	**(a)**	Find the sum of series $\sum_{n=0}^{\infty}\dfrac{2^n-1}{3^n}$.	**03**
	(b)	If $u=\log(x^2+y^2)$, show that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$.	**04**
	(c)	Find all local maxima, local minima and saddle points of the function $f(x,y)=x^3+3xy^2-15x+y^3-15y$.	**07**

OR

Q.4	**(a)**	Discuss Convergence of series $\sum_{n=1}^{\infty}\ln\left(\dfrac{n}{2n+1}\right)$.	**03**
	(b)	Show that $\lim_{(x,y)\to(0,0)}\dfrac{x^4-y^2}{x^4+y^2}$ does not exists.	**04**
	(c)	A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.	**07**
Q.5	**(a)**	Find Jacobian $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ for $x=r\cos\theta, \ y=r\sin\theta$.	**03**
	(b)	Solve the given linear system using Gauss-Elimination method $x_1+x_2-2x_3=1, \ 2x_1-3x_2+x_3=-8, \ 3x_1+x_2+4x_3=7$.	**04**
	(c)	Sketch the region of integration, change the order of integration and evaluate $\int_0^4\int_{\sqrt{x}}^2 \dfrac{1}{y^3+1}dydx$.	**07**

OR

Q.5	**(a)**	Evaluate: $\int_0^2\int_0^{\pi/2} x\sin y\, dydx$.	**03**
	(b)	State Caley-Hamilton theorem. Verify it for matrix $A=\begin{bmatrix}1 & 0\\ 0 & 5\end{bmatrix}$.	**04**
	(c)	Use the transformation $x=u^2-v^2, y=2uv$ to evaluate the integral $\int_0^1\int_0^{2\sqrt{1-x}}\sqrt{x^2+y^2}\,dydx$.	**07**

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