Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:3130005 Date:11-07-2022

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Express the complex number $-\sqrt{3}-i$ in polar form.	**03**
	(b)	Use De Moiver's theorem and find $\sqrt[3]{64i}$.	**04**
	(c)	Verify that $u = 2x - x^3 + 3xy^2$ is harmonic in the whole complex plane and finds it's harmonic conjugate function $v(x, y)$.	**07**
Q.2	**(a)**	Discuss Continuity of the function $f(z)$ at the origin: $f(z) = \begin{cases} \frac{Im(z)}{z}, & z \neq 0 \\ 0 & z = 0 \end{cases}$	**03**
	(b)	1) Define $Log(x + iy)$ 2) Determine $Log(-1 + i\ \ 2)$ 3) Determine all values of $log(1 + i)$	**04**
	(c)	Find the image of the circle $\|z + i\| = 2$ under the transformation $w = \frac{1}{z}$. Also, show the regions graphically.	**07**
		OR	
	(c)	Check whether the function $f(z) = \sin z$ is analytic or not. If so, find its derivative.	**07**
Q.3	**(a)**	Evaluate $\oint_C \frac{\sin z}{(z-\pi)^2} dz$, where C is the circle $\|z\| = 4$	**03**
	(b)	Find the Laurent's series that represent $f(z) = \frac{1}{(z-2)(z-3)}$ in the region $2 < \|z\| < 3$.	**04**
	(c)	Find the residues of the function $\mathrm{f(z)} = \frac{z}{(z+1)^2(z^2-4)}$ at its poles.	**07**
		OR	
Q.3	**(a)**	Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = x$	**03**
	(b)	Evaluate $\oint_C \frac{3z+4}{z^2+2z-3} dz$, where C is $\|z\| = 2$	**04**
	(c)	Using Residue theorem, evaluate the following Integral: $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$	**07**
Q.4	**(a)**	Expand $f(z) = \frac{\sin z}{z^4}$ in Laurent's series about $z = 0$ and identify the singularity.	**03**
	(b)	Solve: $\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 25\frac{\partial^2 z}{\partial y^2} = e^{3x+2y}$	**04**
	(c)	Solve $x^2 p + y^2 q = (x + y)z$	**07**

OR

Q.4	**(a)**	Find the fixed points of the transformation, $w = \frac{z-1}{z+1}$	**03**
	(b)	Solve: $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \cos x$	**04**
	(c)	Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\frac{\partial z}{\partial y} = -2 \sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\frac{\pi}{2}$	**07**
Q.5	**(a)**	Solve $xp + yq = 3z$	**03**
	(b)	Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, where $u(0, y) = 8e^{-3y}$	**04**
	(c)	A tightly stretched string with fixed end points at $x = 0$ and $x = 10$ is initially given by the deflection $f(x) = kx(10 - x)$. If it is released from this position, then find the deflection of the string.	**07**

OR

Q.5	**(a)**	Find complete and singular solution of $z = px + qy + pq$	**03**
	(b)	Using Charpit's method, solve $q = 3p^2$.	**04**
	(c)	A rod of 30 cm long has its ends A and B are kept at 20℃ and 80℃ respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0℃ and kept so. Find the resulting temperature $u(x, t)$from the end A.	**07**