# GUJARAT TECHNOLOGICAL UNIVERSITY
### BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2021

**Subject Code:3140708**                                            **Date:07/09/2021**
**Subject Name:Discrete Mathematics**
**Time:02:30 PM TO 05:00 PM**                          **Total Marks: 70**
**Instructions:**
     1. **Attempt all questions.**
     2. **Make suitable assumptions wherever necessary.**
     3. **Figures to the right indicate full marks.**
     4. **Simple and non-programmable scientific calculators are allowed.**

**MARKS**

**Q.1** **(a)** Among 100 people at least how many of them were born in the same month?      **03**

**(b)** Prove that: $(A \cup B)' \equiv A' \cap B'$.      **04**

**(c)** Define the following:      **07**
1) Composition of functions
2) Monoid
3) Existential Quantifier
4) Partially Ordered Set
5) Boolean Algebra
6) Tree
7) Complete Graph

**Q.2** **(a)** Explain types of a Relation with a suitable example.      **03**

**(b)** Rewrite the following statements using quantifier variables and predicate symbols:      **04**
1) All birds can fly.
2) Some women are genius.
3) There is a student who likes Discrete Mathematics but not Probability and Statistics.
4) Each integer is either even or odd.

**(c)** Determine the validity of the argument given:      **07**
If I study, then I will not fail in Discrete Mathematics.
If I do not play cricket, then I will study.
But I failed in Discrete Mathematics.
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Therefore I must have played cricket.

**OR**

**(c)** Find if the following is a tautology, contradiction or contingency.      **07**
$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

**Q.3** **(a)** Define: Bounded, Distributive and Complemented Lattices.      **03**

**(b)** Find the transitive closure of      **04**
$R = \{(1,2), (3,4), (4,5), (4,1), (1,1)\}$. Where, $A = \{1,2,3,4,5\}$.

**(c)** Let $A$ be a set of factors of positive integer $m$ and relation is divisibility on $A$.      **07**
For $m = 45$, show that POSET $(A, \leq)$ is a Lattice.

**OR**

**Q.3** **(a)** Draw the Hasse diagram of the set $\{1,3,9,18\}$ under partial order relation 'divides' and indicate those which are chains.      **03**

**(b)** Let $X = \{1,2,3,...,7\}$ and $R = \{(x,y): x - y \text{ is divisble by } 3\}$. Show that $R$ is an equivalence relation. Draw the graph of $R$.      **04**

1

**(c)** Solve the recurrence relation:     **07**

$$a_{n+2} - 5a_{n+1} + 6a_n = 2.$$

**Q.4** **(a)** Define group with example. Give an example of a non-abelian group.     **03**

**(b)** Let $H = \{[0], [3]\}$ in $Z_6$ under addition. Find left and right cosets in $< Z_6, +_6 >$.     **04**

**(c)** Prove that $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7.     **07**
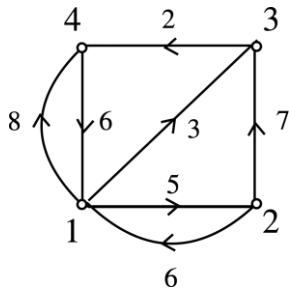
**OR**

**Q.4** **(a)** Define subgroup and group Homomorphism.     **03**

**(b)** Is addition a binary operation on $\{-1,0,1\}$? Justify.     **04**
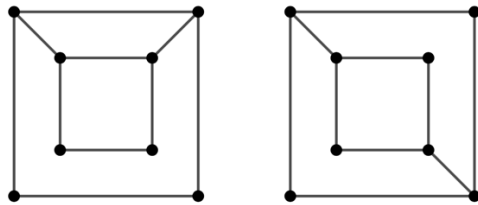
**(c)** Explain Cosets and Lagrange's theorem.     **07**

**Q.5** **(a)** How many nodes are necessary to construct a graph with exactly 8 edges in which each node is of degree 2.     **03**

**(b)** Find the shortest path between each pair of vertices for a simple digraph using Warshall's Algorithm.     **04**



**(c)** Define Isomorphic Graphs. Verify the following graphs are Isomorphic or not (Justify).     **07**



**OR**

**Q.5** **(a)** Define Cyclic graph, Null graph and Strongly connected graph.     **03**

**(b)** Draw a graph which is regular but not bipartite.     **04**

**(c)** For the following set of weights construct an optimal binary prefix code. For each weight in the set, give corresponding code word.     **07**

5, 7, 8, 15, 35, 40

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