Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 2110015 **Date: 01/06/2019**

Subject Name: Vector Calculus & Linear Algebra

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No.1 is compulsory. Attempt any four out of remaining six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)**

(a) **07**

1. The matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is in the form

(a) Row echelon. (b) Reduced row echelon. (c) Both (a) and (b). (d) None.

2. For $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ the $|A^k|$ =_____

(a) 1 (b) 2 (c) 2^k (d) 2^{k-1}

3. If u and v are vectors in a real inner product space, and ||u||=2, ||v||=3, then |<u,v>| ≤ ______

(a) 6 (b) 3 (c) 2 (d) 1.5

4. Which of the following doesn't lie in the space spanned by $\cos^2 x$ and $\sin^2 x$?

(a) 1 (b) 0 (c) Sin x (d) Cos 2x

5. Dimension of the subspace { p(x) ϵ P_2 : p(0) = 0 } of $P_2=\{a+bx+cx^2 : a, b, c \in R\}$ is

(a) 3 (b) 2 (c) 1 (d) 0

6. Which of the following subsets of R^2 is linearly dependent?

(a) {(1,2), (2,1)} (b) {(1,2), (2,1), (1,1)} (c) {(1,2)} (d) None

7. Let T: $R^2 \to R^2$ defined by T(x,y) = (x,0) then Ker (T) =____

(a) Y-axis (b) X-axis (c) Origin (d) None

(b) **07**

1. Which of the following is not an elementary matrix?

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

2. For $\bar{a}$ = (1, −1, 2) , $\bar{b}$ = (1, 3, 1) are vectors of R^3 with Euclidean inner product then $\cos\theta$ =______, where θ is the angle between the two vectors.

(a) 1 (b) 0 (c) − 3 (d) 6

3. Which of the following is not true?

(a) $(AB)^T = B^TA^T$ (b) $(AB)^{-1} = B^{-1}A^{-1}$ (c) $(A^T)^T = A$ (d) $A^T = -A$

4. If A is n×n matrix having rank n−1 then A, A^2, A^3, ……….., A^k,…. have common eigenvalue _____

(a) 1 (b) −1 (c) 0 (d) 2

5. If A is unitary matrix then A^{-1} =____

(a) A (b) A^2 (c) A^T (d) I

6. The dimension of the solution space of x − y = 0 is ____

(a) 0 (b) 1 (c) 2 (d) 3

7. If f(x,y,z) = xyz then Curl (grad f) = ___

(a) 0 (b) x (c) xi+yj+zk (d) xyz

Q.2 (a) Which of the following are linear combination of u = (0, −2, 2) and v = (1, 3, −1)? Justify! (i) (2,2,2), (ii) (0, 4, 5) **03**

(b) Using Gram-Schmidt orthogonalization process find the corresponding orthonormal set to { (1, 1, 1), (0, 1, 1), (0, 0, 1)}. **04**

(c) Using Gauss- Jordan elimination find the inverse of $\begin{pmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -8 \end{pmatrix}$. **07**

Q.3 (a) Find the rank of the matrix and basis of the null space of $\begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$. **03**

(b) Solve the system of linear equations using Gauss elimination method: **04**

$x + y + 2z = 8,\ -x - 2y + 3z = 1,\ 3x - 7y + 4z = 10.$

(c) Show that the set of all real numbers of the form (x, 1) with operations **07**

$(x, 1) + (x', 1) = (x + x', 1)$ and $k(x, 1) = (kx, 1)$ forms a vector space.

Q.4 (a) Determine whether the following are linear transformation or not? **03**

(i) $T: P_2 \rightarrow P_2$, $T(p(x)) = p(x + 1)$,

(ii) $T: P_2 \rightarrow P_2$, $T(a + bx + cx^2) = (a + 1) + (b + 1)x + (c + 1)x^2$.

(b) Which of the following sets of vectors of R^3 are linearly independent? Justify. **04**

(i) { (4, −1, 2), (−4, 10, 2)} (ii) {(−3, 0, 4), (5, −1, 2), (1, 1, 3)}

(c) Find the eigenvalues and bases for the eigenspaces for A^{11}, **07**

$$A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

Q.5 (a) Find basis of kernel and range of $T: R^2 \rightarrow R^2$, defined by **03**

$T(x, y) = (2x - y, -8x + 4y)$

(b) Which of the following are basis of R^3? Justify! **04**

(i) { (1, 0, 0), (2, 2, 0), (3, 3, 3) } , (ii) { (3, 1, −4), (2, 5, 6), (1, 4, 8)}

(c) Let $T: P_2 \rightarrow P_2$, defined by $T(p(x)) = p(3x - 5)$ **07**

(i) Find the matrix of T with respect to the basis $\{1, x, x^2\}$.

(ii) Use the indirect procedure using matrix to compute $T(1 + 2x + 3x^2)$.

(iii) Check the result in (b) by computing $T(1 + 2x + 3x^2)$ directly.

Q.6 (a) Show that $\bar{F} = \frac{(yi - xj)}{x^2 + y^2}$ is irrotational. **03**

(b) Find the directional derivative of $f(x, y, z) = x^2z + y^3z^2 - xyz$ at (1,1,1) in the direction of the vector (−1,0,3). **04**

(c) Using Green's theorem evaluate $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$, where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. **07**

Q.7 (a) Find the work done by $\bar{F} = (y - x^2)\,i + (z - y^2)\,j + (x - z^2)\,k$ over the curve $\mathbf{r}(t) = t\,i + t^2\,j + t^3\,k$; $0 \le t \le 1$, from (0,0,0) to (1,1,1). **03**

(b) Use Cramer's rule to solve: $x + 2z = 6,\ -x + 4y + 6z = 30,\ -x - 2y + 3z = 8.$ **04**

(c) Verify divergence theorem for $\bar{F} = x\,i + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$. **07**