Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 2110014 **Date: 06/06/2019**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Question No.1 is compulsory. Attempt any four out of remaining six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Marks**

(a) **07**

1. For the Jacobian J, value of the $J \cdot J'$ is
(a)1 (b) -1 (3) 0 (4) 2

2. Value of $\frac{dy}{dx}$ for $ax^2 + 2hxy + by^2 = 1\ is$
(a)$\frac{hx+by}{ax+hy}$ (b) $\frac{ax+hy}{hx+by}$ (c) $-\frac{ax+hy}{hx+by}$ (d) $-\frac{hx+by}{ax+hy}$

3. $u = sin^{-1}\frac{x}{y}$ is a homogeneous function of degree
(a) 1/2 (b) 0 (c) 1 (d) -1

4. The curve $r = 2$ is
(a) straight line (b) point at distance '2' on initial line
(c) circle with centre origin and radius 2 (d) cardioid

5. If $x = rcos\theta, y = rsin\theta$,then which is correct?
(a) $r = x^2 + y^2, \theta = \frac{x}{y}$ (b) $r = \sqrt{x^2 + y^2}, \theta = tan\frac{y}{x}$
(c) $r = x^2 + y^2, \theta = tan^{-1}\frac{y}{x}$ (d) $r = \sqrt{x^2 + y^2}, \theta = tan^{-1}\frac{y}{x}$

6. Infinite Sequence $\{1,1,1, \dots\}$ is
(a) convergent (b) divergent (c) oscillatory (d) None of these

7. Infinite Series $1 + 1 + 1 + \cdots is$
(a) convergent (b) divergent (c) oscillatory (d) None of these

(b) **07**

1. Infinite series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - + \cdots$ is
(a) convergent (b) divergent (c) oscillatory (d) None of these

2. Curve $(y - 1)^2 = x - 5$ is symmetric to
(a) X-axis (b) line $y = -x$ (c) line $y = x$ (d) Y- axis

3. $\lim_{x \to 0} \frac{\tan \pi x}{x}$
(a)$\frac{1}{\pi}$ (b) 0 (c) ∞ (d) π

4. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is
(a) ∞ (b) $1/2$ (c) 2 (d) 1

5. The Maclaurin series for the function $(x + 1)^2$ is
(a) $1 + x + x^2$ (b) $1 + 2x + x^2$ (c) $1 + x$ (d) $x + x^2$

6. The straight line $y = 2$ is revolved about x- axis between $0 \ll x \ll 4$. The generated solid is
(a)cone (b) sphere (c) cuboid (d) cylinder

7. For a series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n \to \infty} a_n \neq 0$, then
(a) series is convergent (b) series is divergent
(c) sum of series is finite number
(d) series is conditionally convergent

Q.2	**(a)**	Find the Taylor series for $f(x) = \frac{1}{x}$ at $a = 2$.	**03**
	(b)	Is the series absolutely convergent or conditionally convergent? $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + - \cdots$	**04**
	(c)	(i) Discuss the convergence of the series $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots$	**04**
		(ii) Find the Radius of convergence for the series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.	**03**
Q.3	**(a)**	Evaluate $\lim_{x\to 0} x\log x$	**03**
	(b)	Trace the curve $y^2(a+x) = x^2(a-x),\ \ a > 0.$	**04**
	(c)	Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \ll 1$.	**07**
Q.4	**(a)**	Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.	**03**
	(b)	Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 + z - 9 = 0$ at $(1, 2, 4)$.	**04**
	(c)	(i)Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.	**04**
		(ii) Evaluate $\lim_{x\to\frac{\pi}{2}} (1 - \cos x)^{\tan x}$	**03**
Q.5	**(a)**	If $u = f(x-y, y-z, z-x)$, prove that $u_x + u_y + u_z = 0$.	**03**
	(b)	Find maximum and minimum values. $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$	**04**
	(c)	If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that (i) $xu_x + yu_y = \sin 2u$ (ii) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 2\sin u \cos 3u$	**07**
Q.6	**(a)**	The region between the curve $y = \sqrt{x}, 0 \ll x \ll 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.	**03**
	(b)	Using volume by slicing method, find the volume of a cylinder with radius 'r' and height 'h' .	**04**
	(c)	Evaluate $\iint_R x\, dxdy$; R is triangle $(0,0), (1.0), (1,1)$ using transformations $x = u, y = uv$.	**07**
Q.7	**(a)**	Evaluate $\iint r^3 dr d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.	**03**
	(b)	Evaluate $\int_0^1 \int_0^{1-x} \int_0^{(x+y)^2} x\ \ dzdydx$	**04**
	(c)	Change the order of integration and evaluate. $\int_0^1 \int_{x^2}^{2-x} xy\ dydx$	**07**

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