Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER 2018

Subject Code: 2110015 **Date: 17-05-2018**

Subject Name: Vector Calculus and Linear Algebra

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) Choose the appropriate answer for the following questions. **07**

1. A square matrix whose determinant is non zero is called
(A) Singular (B) non-singular (C) invertible (D) both B and C

2. If A and B are non singular matrices then $(AB)^{-1} =$ ____
(A) $A^{-1}B^{-1}$ (B) AB (C) $B^{-1}A^{-1}$ (D) none of these

3. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then A is in
(A) Row echelon form (B) Reduced Row echelon form (C) both A and B (D) none of these

4. For what values of k does the system $x + y = 2, 3x + 3y = k$ has infinitely many solutions
(A) K=5 (B) k=4 (C) k=6 (D) k=1

5. If in a set of vectors atleast one member can be expressed as a linear combination of the remaining vectors then the set is
(A) Linearly independent (B) Linearly dependent (C) basis (D) none of these

6. If V is any vector space and S be a subset of V then S is called basis for V if
(A) S is Linearly independent (B) S spans V (C) both A and B
(D) S is Linearly dependent

7. For what value of k the vectors u and v are orthogonal where u=(2,1,3) , v=(1,7, k)
(A) K=-3 (B) k=1 (C) k=5 (D) k=2

(b) Choose the appropriate answer for the following questions. **07**

1. The eigen values of a matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ are
(A) 1,4 (B) -1,-4 (C) 1,3 (D) -1,3

2. If A is a nxn size invertible matrix then rank of A is
(A) n-1 (B) n (C) 2n (D) n+1

3. If $\overline{F}$ is solenoidal then

(A) $\nabla \overline{F} = 0$ (B) $\nabla \times \overline{F} = 0$ (C) $\nabla \bullet \overline{F} = 0$ (D) none of these

4. The mapping $T : R^3 \to R^3$ *defined by* $T(x, y, z) = (x, y, -z)$ is called as

(A) Contraction (B) Projection (C) Reflection (D) Rotation

5. The linear transformation $T : V \to W$ is one to one if and only if the nullspace of T consists of only

(A) Identity vector (B) zero vector (C) any non zero vector (D) none of these

6. If $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ then the rank of the matrix A is

(A) 1 (B) 2 (C) 0 (D) 4

7. Let A be a skew-symmetric matrix then

(A) $a_{ij} = a_{ji}$ (B) $a_{ij} = -a_{ji}$ (C) $a_{ii} = 0$ (D) both B and C

Q.2 **(a)** Find the unit vector normal to the surface $xy^3z^2 = 4$ *at* $(-1, -1, 2)$ **03**

(b) Express the matrix $A = \begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as the sum of a symmetric and skey-symmetric matrix. **04**

(c) **Investigate for what values of λ and μ the equations** **07**

$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ **have**

(1) No solution (2) a unique solution (3) infinite number of solutions

Q.3 **(a)** Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **03**

(b) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by Gauss Jordan Method **04**

(c) **For the basis $S = \{v_1, v_2, v_3\}$ of R^3 where** **07**

$v_1 = (1,1,1), v_2 = (1,1,0), v_3 = (1,0,0)$ Let $T : R^3 \to R^2$ be the linear transformation such that

$T(v_1) = (1,0), T(v_2) = (2,-1), T(v_3) = (4,3)$ find a formula for $T(x_1, x_2, x_3)$ and then use the formula to find $T(4,3,-2)$

Q.4	**(a)**	Determine whether the vector $v = (-5, 11, -7)$ is a linear combination of the vectors $v_1 = (1, -2, 2), v_2 = (0, 5, 5), v_3 = (2, 0, 8)$	**03**
	(b)	Solve the linear system $x + y + z = 4,\ -x - y + z = -2,\ 2x - y + 2z = 2$ by gauss elimination method.	**04**
	(c)	Let R^3 have the Euclidean inner product. Use the gram schmidt process to transform the basis (u_1, u_2, u_3) in to Orthonormal basis where $u_1 = (1, 0, 0), u_2 = (3, 7, -2), u_3 = (0, 4, 1)$	**07**
Q.5	**(a)**	Find the eigen values and corresponding eigen vectors of $A = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$	**03**
	(b)	Let $A = \begin{bmatrix} -2 & 3 \\ 1 & -2 \\ 1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ then find the least squares solutions to AX=b	**04**
	(c)	**Let** $T : R^3 \to R^3$ **be a linear operator and** $B = (v_1, v_2, v_3)$ **a basis for** R^3**. Suppose that** $T(v_1) = (1,1,0), T(v_2) = (1,0,-1), T(v_3) = (2,1,-1)$ **then** **(1) Is (1,2,1) in R(T) ? (2) Find a basis for R(T).**	**07**
Q.6	**(a)**	Find the work done by the force $\overline{F} = (3x^2 - 3x)i + 3zj + k$ $along$ the straight line $ti + tj + tk, 0 \le t \le 1$..	**03**
	(b)	Check whether the vectors (2,-3,1), (4,1,1),(0,-7,1) is a basis for R^3	**04**
	(c)	**Verify Green's Theorem for** $\overline{F} = (x - y)i + xj$ and C is $x^2 + y^2 = 1$	07
Q.7	**(a)**	Find the directional derivative of $4xz^2 + x^2yz$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$	**03**
	(b)	Show that $\overline{F} = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative and find the potential function.	**04**
	(c)	Let $V = \{(a, b) / a, b \in R\}$ and let $v = (v_1, v_2), w = (w_1, w_2)$ then define $(v_1, v_2) + (w_1, w_2) = (v_1 + w_1 + 1, v_2 + w_2 + 1)$ and $c(v_1, v_2) = (cv_1 + c - 1, cv_2 + c - 1)$ then verify that V is a vector space.	**07**

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