Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER 2018

Subject Code: 2110014 **Date: 21-05-2018**

Subject Name: Calculus

Time: 02:30 pm to 05:30 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. The sequence $\left\{\frac{\cos 2n}{n}\right\}$ converges to

(a) 1 (b) 0 (c) 2 (d) -1

2. Sum of the series $\sum_{n=0}^{\infty}\frac{4}{2^n}$ is

(a) 4 (b) 2 (c) 8 (d) 16

3. The coefficient of x^4 in the expansion of $\cos x$ is

(a) $\frac{1}{4!}$ (b) $-\frac{1}{4!}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

4. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x =$ ____________

(a) 0 (b) 1 (d) e (d) ∞

5. The curve $x^3 + y^3 = 3xy$ is symmetric about

(a) x-axis (b) y-axis (c) line y = x (d) origin

6. Asymptote parallel to x-axis of the curve $3x^3 + xy^2 + xy = 0$ is

(a) y=3 (b) y=1 (c) y=0 (d) not possible

7. The curve $r^2 = a^2 cos2\theta$ is not symmetric about

(a) initial line (b) line $\theta = \frac{\pi}{4}$ (c) line $\theta = \frac{\pi}{2}$ (d) pole

(b) **07**

1. If $u = ytan^{-1}(x/y) + xcot^{-1}(y/x)$, then $xu_x + yu_y$=_________

(a) 0 (b) u (c) 2u (d) 3u

2. For an implicit function $f(x,y) = c,$ the value of $\frac{dy}{dx}$ is

(a) $\frac{f_x}{f_y}$ (b) $\frac{f_y}{f_x}$ (c) $-\frac{f_x}{f_y}$ (d) $-\frac{f_y}{f_x}$

3. $\lim_{(x,y)\to(0,1)}\frac{y^2tan^{-1}x}{x}$ =_______________

(a) 0 (b) 1 (c) -1 (d) ∞

4. $\int_0^1\int_0^y e^{x/y}\,dxdy$ =__________

(a) $\frac{e-1}{2}$ (b) $e-1$ (c) e (d) $e+1$

5. If $u = x - y$ and $v = x + y$, then the value of $J = \frac{\partial(x,y)}{\partial(u,v)}$ is ______

(a) 1 (b) -1 (c) 1/2 (d) 1/4

6. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent when

(a) $p=1$ (b) $p<1$ (c) $p>1$ (d) $p=0$

7. The series $\sum_{n=1}^{\infty}\frac{n-1}{n+1}$ is__________

(a) divergent
(b) convergent and sum 1
(c) convergent and sum 2
(d) none of these

Q.2	**(a)**	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n+1}{n^3-3n+2}$	**03**
	(b)	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{4^n+1}{5^n}$	**04**
	(c)	Evaluate (i) $\lim_{x\to 0}\frac{2x-x\cos x-\sin x}{2x^3}$; (ii) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.	**07**
Q.3	**(a)**	If $u=\log(x^2+y^2)$, verify that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$.	**03**
	(b)	If $u=r^m$, prove that $u_{xx}+u_{yy}+u_{zz}=m(m+1)r^{m-2}$, where $r^2=x^2+y^2+z^2$.	**04**
	(c)	Find the maxima and minima of the function $f(x,y)=x^2y-xy^2+4xy-4x^2-4y^2$.	**07**
Q.4	**(a)**	If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$.	**03**
	(b)	Evaluate $\iint(6x^2+2y)dxdy$ over the region R bounded between $y=x^2$ and $y=4$.	**04**
	(c)	Change the order of integration and evaluate $\int_0^4\int_{x^2/4}^{2\sqrt{x}}dydx$.	**07**
Q.5	**(a)**	If $z=xy^2+x^2y,\ x=at^2,\ y=2at$, find $\frac{dz}{dt}$.	**03**
	(b)	If $u=e^{x^2+y^2-xy}$, then prove that (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u\ln u$; (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2u\ln u(2\ln u+1)$.	**04**
	(c)	(i) Check the absolute and conditional convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{n}}$. (ii) Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(-5)^n x^n}{n!}$.	**07**
Q.6	**(a)**	Evaluate $\iint xdA$, over the region R bounded by the parabolas $y^2=4x$ and $x^2=4y$.	**03**
	(b)	Expand $\cos\left(\frac{\pi}{4}+x\right)$ in powers of x by Taylor series. Hence find the value of $\cos 46°$.	**04**
	(c)	Evaluate $\int_0^4\int_{y/2}^{(y/2)+1}\frac{2x-y}{2}dxdy$ by applying the transformations $u=\frac{2x-y}{2}, v=\frac{y}{2}$.	**07**
Q.7	**(a)**	Evaluate the improper integral $\int_{-\infty}^{\infty}\frac{1}{1+x^2}\,dx$.	**03**
	(b)	Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=2, x=0$ about the line $y=2$.	**04**
	(c)	Trace the curve $y^2(a-x)=x^3,\ a>0$.	**07**

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