DISCRETE-TIME SLIDING MODE CONTROL FOR NETWORKED CONTROL SYSTEM

A Thesis submitted to Gujarat Technological University

for the Award of

Doctor of Philosophy

in

Instrumentation and Control Engineering

By

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under the supervision of

Dr. Axaykumar J Mehta

GUJARAT TECHNOLOGICAL UNIVERSITY, AHMEDABAD

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ABSTRACT

Networked control systems (NCSs) are traditional feedback control loops closed through a real time communication network. That is, in networked control systems, communication networks are employed to exchange information (reference input, plant output, control input, etc...) between control system components (sensors, controllers, actuators, etc...). NCS has become popular in the field of control due to its distinct advantages such as low cost, reduced weight, simple installation and maintenance, resource sharing and high reliability. As a result, NCS have great potential in industrial applications such as manufacturing plants, smart grid, haptic collaboration, vehicles, aircrafts, robotics, spacecrafts and many others. NCS generally possess a dynamic nature which results in various challenges for researchers in terms of random time delay, packet loss, multiple packet loss, packet disordering, resource allocation and bandwidth sharing. It is well known that the performance of NCS is significantly deteriorated due to these communication uncertainties. If these challenges are not handled properly, they may result in degradation of the system's performance. Among all these issues, time delay and packet loss are considered to be crucial issues in NCS that degrades the stability and control performance of closed-loop control systems.

The delays may be constant, time-varying, and in most cases, random. The nature of network delay mainly depends on the configuration of the communication medium. If the communication medium is configured using lease line concept then the delays are always deterministic in nature. And whenever the communication medium is shared by large number of devices then the delays are random in nature. It is worth to mention here that, the amount of time required for the data packets to travel from sensor to controller and controller to actuator is defined as total network delay. The controller mainly suffers from sensor-to-controller delay. When such delay is transformed into discrete-time domain it mostly possesses non-integer type of values. Such network delays in discrete-time domain are defined as fractional delays which may be either deterministic or random in nature. So, it is important to compensate the effect of deterministic as well random fractional delay in discrete-time domain at each sampling instant.

Further, as mentioned above there are also possibilities of packet loss during the transmission of data packets from sensor to controller as well as controller to actuator. The packet loss takes place due to heavy network load, network congestion and node competition. In discrete-time domain, the network-induced delay greater than one sampling time is also considered as packet loss. The nature of delay and packet loss is mainly dependant on configuration of network medium.

In recent years, many algorithms have been studied for the stability analysis and control design of NCS in discrete-time domain that includes state feedback controller, H_∞, Model predictive controller and sliding mode controller etc... Among them Sliding Mode Controller is one of the robust control algorithms because of its invariance properties to parameter variation and uncertainties.

This thesis presents novel algorithms for designing Discrete-time Sliding Mode Controller (DSMC) for NCS having both types of fractional delays i.e. deterministic and random alongwith different packet loss conditions i.e. single packet loss and multiple packet loss.

KEYWORDS: Networked control system, Discrete-time sliding mode control, Fractional delay, Packet loss, Multi-rate output feedback.

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I want to dedicate this thesis to my late father, who always wished to see me a doctorate. I am blessed and honoured to be able to fulfil his dream. The major credit of my thesis completion belongs to my dearest mother. Without the inspiration, drive and support that she gave me, I could not have achieved all that I have. Lastly, I thank all my family members who have helped me socially and psychologically.

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ABBREVIATIONS

NCS	Networked Control System
CAN	Controller Area Network
LAN	Local Area Network
WAN	Wide Area Network
MAN	Metropolitan Area Network
PAN	Personal Area Network
FIFO	First In First Out
TOD	Try-one-discard
GSM	Gain Schedule Middleware
RTT	Round Trip Time
WSN	Wireless Sensor Networks
ZOH	Zero-Order Hold
RMPC	Robust Model Predictive Control
LQG	Linear Quadratic Gaussian
SMC	Sliding Mode Control
VSC	Variable Structure Control
LQR	Linear Quadratic Regulator
DSMC	Discrete-time Sliding Mode Control
SISO	Single Input Single Output
LTI	Linear Time Invariant
CSMA/CD	Carrier Sense Multiple Acces Collision Detection
MROF	Multi-rate Output Feedback
DNSMC	Discrete-time Networked Sliding Mode Control

LIST OF SYMBOLS

C_M	centeralized controller
$x(t)$	plant state vector in continuous-time domain
$u(t)$	control input signal in continuous-time domain
$d(t)$	slow time varying disturbance signal in continuous-time domain
$y(t)$	output signal in continuous-time domain
$x(k)$	plant state vector in discrete-time domain
$u(k)$	control input signal in discrete-time domain
$d(k)$	disturbance signal in discrete-time domain
$y(k)$	output signal in discrete-time domain
A	system matrix in continuous-time domain
B	control input matrix in continuous-time domain
C	output matrix
F	system matrix in discrete-time domain
G	control input matrix in discrete-time domain
a_1, a_2, M_s, c	constants
$s(t)$	sliding surface in continuous-time domain
$s_d(k)$	priori function
k_s, ι, ψ	user defined constant
δ_0	positive offset
p_s, k'	positive integer
k_t	user defined gain
k_t^+, k_t^-	lower and upper bounds coefficients
$\hat{S}$	model uncertainty
S_1, S_2	mean and deviated value of $\hat{S}$
S_u, S_l	upper and lower bounds of $\hat{S}$
τ_t	total delay which includes system and network delay in continuous-time domain
τ	total network delay in continuous-time domain
τ_{sc}	sensor to controller delay in continuous-time domain

τ_{ca}	controller to actuator in continuous-time domain
h	sampling interval
τ'	deterministic total network fractional delay
τ'_{sc}	deterministic sensor to controller fractional delay
τ'_{ca}	deterministic controller to actuator fractional delay
d_l	lower bound of disturbance
d_u	upper bound of disturbance
τ_p	system delay in continuous-time domain
τ_{sp}	sensor processing delay
τ_{cp}	controller computational delay
τ_{ap}	actuator processing delay
ν	fractional part of delay
l	order of approximation
δ	signal transmission delay
$s(k)$	sliding variable in discrete-time domain
C_s	sliding gain
α	parameter that depends on sensor to controller fractional delay
Q, R	matrices of appropriate dimensions
ϵ, q	user defined constants
sgn	signum function
d_1	mean value of disturbance
d_2	deviated value of disturbance
d_s	compensated disturbance signal
$\eta, \beta, \rho, \gamma$	smallest parameter constant
V_s	lyapunov function
Φ, κ	stability parameters
α'	parameter computed based on actuator to controller fractional delay
u_a	compensated control signal at actuator side
T_s	settling time
τ_{CAN}	amount of network delay in CAN medium
τ_{ETHER}	amount network delay in Switched Ethernet medium
$\theta(s)$	output of the system (position)
V_m	input to the system

J_m	rotor inertia
R_m	terminal resistance
K_m	motor back emf constant
τ_r	total random network delay in continuous-time domain
$\hat{\tau}$	total random network fractional delay in discrete-time domain
τ_{rsc}	random sensor to controller delay in continuous-time domain
τ_{rca}	random controller to actuator delay in continuous-time domain
$\hat{\tau}_{sc}$	random sensor to controller fractional delay in discrete-time domain
$\hat{\tau}_{ca}$	random controller to actuator fractional delay in discrete-time domain
$\hat{\tau}_l$	lower bound of random fractional delay
$\hat{\tau}_u$	upper bound of random fractional delay
$\bar{\alpha}, \bar{\beta}$	probability of state and control data packet loss
$x_c'(k)$	commuicated state variable over the network
$\{d_1, d_2, .., d_q\}$	values in a finite set
β_v	positive scalar quantity
$\alpha(k), d_v$	stochastic variables
$E\{d_v\}$	expectation of stochastic variable d_v
w	number of trials
λ	average number of events per interval
e	Euler's constant
ς, γ'	random parameter generated using Thiran Approximation
$E\{\alpha(k)\}$	expectation of stochastic variable $\alpha(k)$
$u_c(k)$	communicated control signal over the network
Γ	stability parameter computed using packet loss and random fractional delay
ϕ	sampling rate of control input signal
ζ	sampling rate of output signal
F_ϕ	system matrix sampled at ϕ sampling interval
G_ϕ	control input matrix sampled at ϕ sampling interval
Λ	positive integer
F_ζ	system matrix sampled at ζ sampling interval
G_ζ	control input matrix sampled at ζ sampling interval
$\hat{x}$	estimated state variable computed using multirate output feedback approach
y_k	output stack

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CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

1.1 Introduction

1.1.1 Overview of Network Control System

The advent of communication networks, introduced the concept of remotely controlling a system, which gave birth to Networked Control Systems (NCSs). The classical definition of NCSs can be as follows: When a traditional feedback control system is closed via a communication channel, which may be shared with other nodes outside the control system, then the control system is called a Network Control System (NCS) (Ling *et al.*, 2007). An NCS can also be defined as a feedback control system wherein the control loops are closed through a real-time network (Gupta and Chow, 2010; Asif and Webb, 2015). The defining feature of an NCS is that information (reference input, plant output, control input, etc...) is exchanged using a network among control system components (sensors, controllers, actuators, etc...). Figure (1.1) shows the conceptual model of the NCS. The networked medium can be wired or wireless depending on the type of the applications. In NCS, when any form of data that is transmitted through wires then such medium is called as wired network while when any form of data that is transmitted without use of electrical conductor then such medium is called as wireless network medium. The main advantage of using wired medium is data security, while the main advantage of using the wireless network medium is to get rid of wires when NCS architecture is complex. In NCS the wired communication is carried out through CAN, Switched Ethernet, Ethernet, Profibus and Profinet networked medium while wireless communication is done through Wirelss LAN, Wireless PAN, Wireless MAN or Wireless WAN (Asif and Webb, 2015).

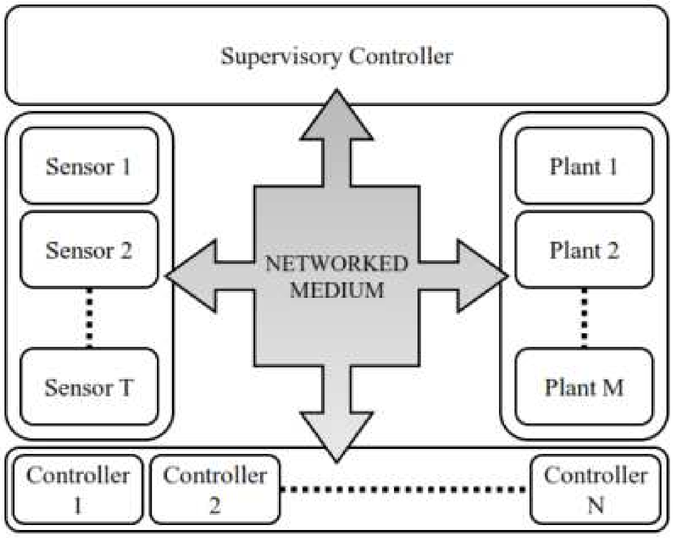


Figure 1.1: Conceptual model of NCS
(Gupta and Chow, 2010)

1.1.2 Structure of Network Control System

In general there are two major types of control systems that utilize communication networks: (1) shared-network control system and (2) remote control system. Figure (1.2) shows the architecture of shared network control system. It can be noticed that using shared-network control system the transfer of information from sensors to controllers and control signals from controllers to actuators can greatly reduce the complexity of connections and provides more flexibility in installation, ease of maintenance and troubleshooting. Moreover, it also provides the communication among control loops [Gupta and Chow (2010), Zhang *et al.* (2013*b*), Zhao *et al.* (2015)]. This feature is extremely useful when a control loop exchanges information with other control loops to perform more sophisticated controls, such as fault accommodation and control. Similar structures for network-based control have been applied to automobiles and industrial plants. The other control system that utilizes the network medium is remote control system. In remote control system, the place where central controller is installed is called a local site and the place where plant is installed is called a remote site. The data transfer between local site and remote site is done through communication network. Sometimes the remote control system is also defined as tele-operation control system. There are two general approaches to design an NCS using remote control system: (i) hierarchical structure and (ii) direct structure. In hierarchical structure there are several subsystems that are connected to central controller through communication network. Each subsys-

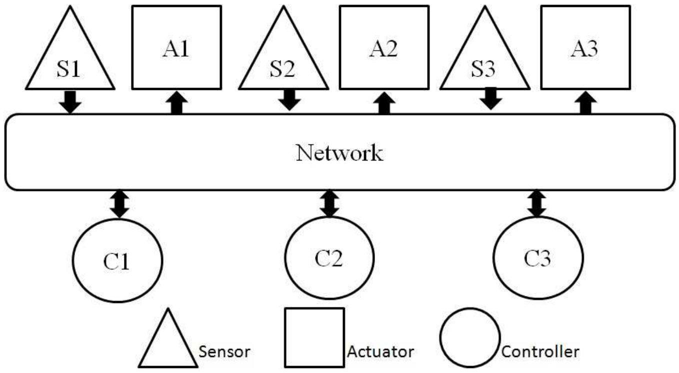


Figure 1.2: Shared structure of NCS
(Gupta and Chow, 2010)

tem contains sensor, actuator, and controller by itself as depicted in Figure (1.3). In this case, a subsystem controller receives a set point from the central controller C_M. The subsystem then tries to satisfy this set point by itself. The sensor data or status signal is transmitted back via network to the central controller. Figure (1.4) shows the

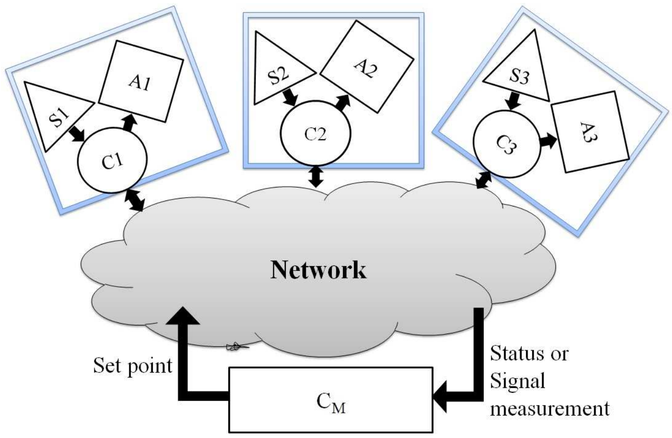


Figure 1.3: Hierarchical structure of NCS
(Gupta and Chow, 2010)

block diagram of direct structure. In this case, the sensor and actuator are attached to a plant, while a controller is separated from the plant by a network connection. The sensor transmits the signal to the controller through the network and controller sends back the processed control signal to the plant via actuator through the network. This type of configuration is used mainly in the process industries and haptic surgery. Many complex network control system use the combination of both the structures defined as hybrid structure.

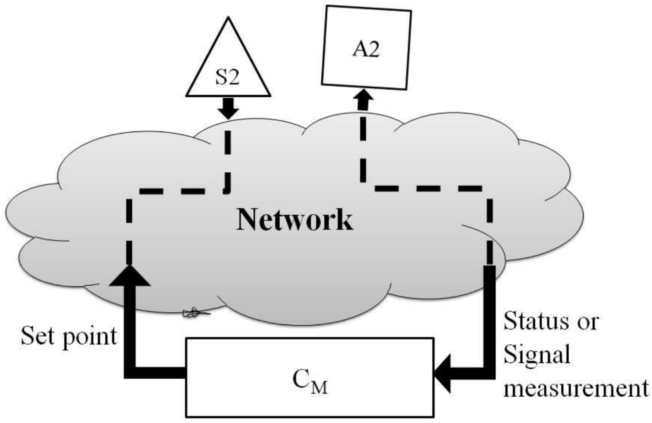


Figure 1.4: Direct structure of NCS
(Gupta and Chow, 2010)

1.1.3 Concerns in Network Control System

The presence of communication medium in network control system leads to several natural issues such as:

- **Time delay**: In network control system time delay is one of the cruicial issue. The time required for the data to travel within the network is defined as time delay. The nature of time delay depends on the various factors such as network configuration, distance of communication between plant and controller, baud rate, network characteristics and network topology. The time delay can affect the performance of the system in all the structure of NCSs (shared, hierarchical and direct).

- **Packet Loss**: The next serious issue that affect the performance of NCS is packet loss. The packet loss is mainly caused due to congestion, network traffic and jitter problems. Whenever the data transmitted from sensor or controller through the network and fails to reach the destination then such condition is defined as packet loss condition. In NCS there are two types of packet loss: (i) single packet loss and (ii) multiplepacket loss. In some of the models of NCS the packet loss situation are interlinked with time delay parameter. The packet loss situation occurs in all the structure of NCSs (shared, hierarchical and direct).

- **Packet disorder**: The packet disorder isssue is generally caused in wireless NCS due to heavy traffic, congestion or jitter. In wireless NCS, the communications takes place in the form of small packets. So, in order to have secure communication each packet is provided with a unique identification number in the header. While transmission if any packet is loss and fails to reach at the destination packet disorder situation takes place. If these disorder is not corrected then it severely affects the performance of the system. This situation takes place in all the structure of NCSs connected wirelessly.

- **Bandwidth Sharing**: This issue mainly occurs in the shared structure or hierarchical structure of NCSs. Both these structures provides the flexiblity of connecting large number of devices (such as plant, controller, sensor and actuator) through a common network medium. So, as the number of devices increases the bandwidth sharing is also increased which in turn causes lesser transmission speed, congestion problem, jittering problem or networked traffic problem. This may cause further instability in the system.

- **Security**: The security issue is one of the major concern in NCS when the communication is done without wires. In wireless communiction there are chances of hacking due to which the false data is generated at the controller side and may cause the instability in the system. This issue needs to be handled very appropriately when the communication is done through shared structure or hierarchical structure of NCS.

1.1.4 Advantages and Applications of NCS

For many years now, data networking technologies have been widely applied in industrial and military control applications. These applications include manufacturing plants, automobiles, and aircraft. Connecting the control system components in these applications, such as sensors, controllers, and actuators, via a network can effectively reduce the complexity of systems, with nominal economical investments. Furthermore, network controllers allow data to be shared efficiently. It is easy to fuse the global information to take intelligent decisions over a large physical space. They eliminate unnecessary wiring. It is easy to add more sensors, actuators and controllers with very little cost and without heavy structural changes to the whole system. Most importantly, they connect cyber space to physical space making task execution from a distance easily accessible. These systems are becoming more realizable today and have a lot of potential applications, including space explorations, terrestrial exploration, factory automation, remote diagnostics and troubleshooting, hazardous environments, experimental facilities, domestic robots, automobiles, aircraft, manufacturing plant monitoring, nursing homes or hospitals, tele-robotics, smart grid etc....

1.2 Literature Survey

Due to its distinct advantages NCS has become popular in the field of control for industrial applications. Also NCS has become an active research topic among international researchers fraternity due to its wide applications. NCS generally possess a dynamic nature which results in various challenges for researchers in terms of random time delay, packet loss, multiple packet loss, packet disordering, resource allocation and bandwidth sharing. If these challenges are not handled properly, they may result in degradation of the system's performance. Among these challenges, time delay and packet loss are considered to be crucial issues in NCS that causes potential instability.

The next section represents the concise literature survey regarding the compensating the effects of network delay and packet loss in continuous-time domain and discrete-time domain.

1.2.1 Continuous-time domain

Various researchers (Luck and Ray (1990*a*,*b*), Luck and Ray (1990*c*, 1994), Chan (1995), Yu *et al.* (2000), Kim *et al.* (2002), Montestruque and Antsaklis (2003), Yue *et al.* (2004), Yang (2006), Godoy and Porto (2010), Li *et al.* (2010), Vardhan and Kumar (2011), Urban *et al.* (2011)) have laid their sincere efforts for designing different control algorithms that compensates the effect of network delay. In the early stages of NCS, when modelling of random time delay was difficult to obtain, the most appropriate approach was to treat the random time delay as constant which are called as deterministic delays. Luck and Ray (1990*a*,*b*) introduced the concept of compensating the time delay in continuous-time domain. They compensated the effect of time delay by introducing the receiver buffer at the controller and actuator side. The size of the buffer was equal to sensor to controller delay and controller to actuator delay. The proposed methodology was tested under IEEE 802.4 network test bed considering the deterministic types of delays. Later on (Luck and Ray, 1990*c*, 1994) designed predictor-based compensator in which observer was designed to estimate the plant states and predictor was used to predict the control sequences based on the past input signals. The FIFO buffer was set at the controller side and actuator side that stores the past output measurements as well as control measurements. The size of the buffer was set according to the upper bounds of sensor to controller delay and controller to actuator delay. They also tested the efficacy of the proposed algorithm on IEEE 802.4 networked medium. Chan (1995) designed conventional form of memory feedback controller based on delay compensation method. Yu *et al.* (2000) designed multiple step delay compensator for NCS in the presence of dynamic noise and measurement noises. Kim *et al.* (2002) modelled an NCS as a switched system with constant input delays and derived the sufficient conditions for the system stability using piecewise continuous Lyapunov methods.

Montestruque and Antsaklis (2003) designed state feedback controller that compensates the effect of deterministic network delays in continuous-time domain. Yang (2006) proposed the state feedback controller in the presence of network delays in continuous-time domain. They proposed ZOH model at controller and actuator side to compensate the effect of network delay. They also assumed that the sensor is time driven device while actuator is event driven device. Godoy and Porto (2010) designed PID controller to compensate the network delay and validate the feasibility of controller through DC motor as plant and CAN (Control Area Network) bus as networked medium. Li *et al.* (2010) designed a method for internet-based network control system in a dual rate configuration to achieve load minimization and dynamic performance specifications. The remote PID controller was design which regulates the output according to desirable reference and adopts the lower sampling rate to reduce the load on the network. The performance of the system was validated for fixed network delays.

In view of this, an increasing number of researchers began to investigate different control methodologies for NCSs with random or time varying delays. Zhang *et al.* (2001) proposed the stability criteria for NCS having network delays shorter as well as longer than sampling interval. They also proposed state feedback controller using conventional estimator technique that compensates the effect of network delays having time varying nature. Similarly, Walsh *et al.* (2002) proposed the mathematical model of NCS considering time varying network induced delay. They derived the stability criteria for general NCS in continuous-time domain based on TOD (try-once-discard) algorithm. Yue *et al.* (2004) designed state feedback controller in the presence of time varying network delays. They assumed that the network delays are lesser than sampling interval. Tipsuwan and Chow (2004) proposed the concept of external gain scheduling via GSM. The GSM was used to adjust the controller gains externally at the controller output with respect to the current network traffic conditions without interrupting the internal design of controller. The network delays in the forward channel and feedback channel were modelled using RTT approach. Ji and Kim (2005) proposed state feedback control with estimator to compensates the effect of time varying network delay in the presence of matched uncertainty. They tested the efficacy of the proposed controller using Ethernet as a network medium. Ma and Zhao (2006) derived the stability criteria for closed loop NCS using the average dwell time approach and piecewise Lyapunov function method. They designed state feedback controller with estimator that takes care of sensor to controller delay. Peng and Yue (2006) designed the state feedback controller for NCS considering time varying network delay in the states and matched uncertainty. Gao *et al.* (2007) proposed a new time delay system approach which contains multiple successive delay components in the plant states and based on that they designed the H_∞ controller to overcome the effect of these state delays. Liu *et al.* (2007) designed network predictive controller to overcome the effects of random network delay in continuous-time domain. The effects of random delays were compensated through network delay compensator placed on the actuator side. The network delay chooses the control input values from the control latest prediction sequence. Cuellar *et al.* (2007) proposed an observer based predictor using the Pade approximation technique for time lag processes. Sun and Xu (2009) modelled the random time delays using stochastic approach in continuous-time domain. They used Markov jump linear systems approach to model sensor to controller random delay while controller to actuator delay was assumed to be constant. Yuhong and Yeguo (2010) designed state feedback controller considering time varying network delay in the states and proved the closed loop NCS stability using LMI approach. Ono *et al.* (2010) designed a state feedback controller based on a modified Smith predictor which stabilized the plant in the presence of dead time. Vardhan and Kumar (2011) used smith predictor algorithm to compensate the effects of time varying network delays in continuous-time domain. Urban *et al.* (2011) studied the effect of network delays in wired and wireless networked medium using PID controller. They used CAN protocol for the wired communication and Zigbee protocol for wireless networked medium. Similarly, Ridwan and Trilaksono (2011) designed the H_∞ state feedback controller assuming all states variable are measurable in the presence of time-varying network delays. Vallabhan *et al.* (2012) have used the analytical framework approach for compensation of random time delay and packet loss. Hikichi *et al.* (2013) worked on continuous-time delay compensation using predictors and disturbance observer for designing a PID controller. Hu *et al.* (2013) designed a sliding mode intermittent controller for bidirectional associative memory (BAM) using neural networks with delays. Cac *et al.* (2014) used a pole placement method for compensating the time delay in the continuous-time domain. The algorithm was designed for the CAN type deterministic networked medium. Recently, Yi *et al.* (2014) solved the time delay problem by using the Smith predictor algorithm. The method was verified over wireless sensor networks (WSN) connected between the controller output and plant input. Recently, Khanesar *et al.* (2015) modelled the random time delays using a uniform probability distribution function in continuous-time domain. Saravanakumar *et al.* (2016) proved the stability using a Markovian Jump approach for neural networks having varying time interval delays.

1.2.2 Discrete-time domain

Like continuous-time domain, many researchers (Jacovitti and Scarano (1993), Nilsson *et al.* (1998), Shousong and Qixin (2003), Zhivoglyadov and Middleton (2003), Yue *et al.* (2005), Zhao *et al.* (2008), Gou (2009), Xiong and Lam (2009), Li *et al.* (2014), Guo *et al.* (2014), Yao *et al.* (2014), Argha *et al.* (2015)) have also tried to focus their work in discrete-time domain. Jacovitti and Scarano (1993) proposed various time delay estimation techniques for discrete-time systems. Nilsson *et al.* (1998) used stochastic approach to design state feedback controller for time varying network delays in discrete-time domain. Similarly, Shousong and Qixin (2003) also used stochastic approach for designing optimal controllers for NCS. They assumed that the random network delays are greater than sampling interval. Zhivoglyadov and Middleton (2003) proposed state feedback observer technique for linear network control system to compensate the effect of random delays. Yue *et al.* (2005) provided the model of NCSs with random network induced delay in discrete-time domain. They designed H_∞ controller to compensate the effect of random delays in the presence of matched uncertainty. Zhao *et al.* (2008) designed integrated predictive controller for networked control system. The predictive controller is applied to generate the control predictions for each delayed sensing data and previous control information. They also designed the time delay compensator at actuator side that actively compensates the forward channel delay when control action is taken. Gou (2009) designed the state feedback controller in discrete-time domain based on variable-period sampling approach for random network delays in NCS. Xiong and Lam (2009) introduced the concept of ZOH model at controller and actuator side that compensates the effect random network delays in discrete-time domain. The proposed ZOH model has an capability of choosing the newest control input. Yang *et al.* (2010) proposed discrete-time sliding mode observer that estimates the random delay and compensates its effect in the presence of matched uncertainty. Li *et al.* (2014) designed a sliding mode predictive control for compensation of delay in a networked control system using a Kalman Predictor. They considered networked delays are random in nature with an integral multiple of sampling interval. Guo *et al.* (2014) considered the state estimation problem for wireless NCS. The sliding mode observer was designed to solve the state estimation problem considering stochastic uncertainty and time delay. Yao *et al.* (2014) designed a robust model predictive control (RMPC) and state observer for a class of time varying systems under input constraints such as matched uncertainty. Argha *et al.* (2015) designed stochastic type sliding mode controller that compensates the effect of random networked delay with values lesser than sampling interval.

Various researchers (Nilsson *et al.* (1998), Zhivoglyadov and Middleton (2003), Zhang *et al.* (2005), Yang *et al.* (2006), Gou (2009), Shi and Yu (2009), Dong and Kim (2012), Ge *et al.* (2013), Argha *et al.* (2015)) have also laid their sincere efforts to model random time delays in last decade. Among them Nilsson *et al.* (1998) introduced the time stamp technique to model the random time varying networked delay. Shousong and Qixin (2003) used stochastic approach to model the random network delays. Zhang *et al.* (2005), Gou (2009) as well as Dong and Kim (2012) modelled random time delay using the concept of Markov Chain process in discrete-time domain. They have used two state Markov chain model to describe sensor to controller delay and controller to actuator delay. While, Yang *et al.* (2006) modelled random networked delay using Bernoulli's distributed white sequence approach. Shi and Yu (2009) modelled random delays using Markov chain process and designed output feedback controller to handle the effects of random delay. Ge *et al.* (2013) used an independent and identically distributed approach to model the time varying networked delay and proposed state feedback controller. Recently, Argha *et al.* (2015) proposed Bernoulli's white sequence approach for modelling the random time delay and proposed sliding mode controller in the presence of random time delay and matched uncertainty.

1.2.3 Packet losses

As mentioned above, there are also possibilities of packet loss during the transmission of data packets from sensor to controller as well as controller to actuator. The packet loss takes place due to heavy network load, network congestion and node competition. In NCS there are two types of packet losses (i) single packet loss and (ii) multiple packet loss. The single packet loss situation generally occurs when the communication of data transfer is done over a shorter distance and multiple packet loss situations generally occurs when communication of data transfer is done over a longer distance. In the research works (Zhang *et al.* (2001), Yue *et al.* (2005), Zhang *et al.* (2005), Yang *et al.* (2006), Hespanha *et al.* (2007), Gupta *et al.* (2007), Wu and Chen (2007), Shi and Yu (2009), Zhu and Yang (2009), Niu and Ho (2010), Dong and Kim (2012), Ge *et al.* (2013), Wen and Gao (2013), Li *et al.* (2014), Khanesar *et al.* (2015), Argha *et al.* (2015), Song *et al.* (2016)), mathematical model is proposed assuming that the packet loss within the communication medium occurs when network delay is greater than sampling interval. Zhang *et al.* (2001) consider the deterministic single packet loss model with packet dropouts occurring at an asymptotic rate. Yue *et al.* (2005) designed single and multiple packet loss model in context with random network delays. They assumed that whenever the controller and actuator are not updated the data packet loss takes place for that sampling interval. Zhang *et al.* (2005) considered the packet loss in correspondence with random time delay model. They also made a generalized assumption that when the delays are greater than sampling interval the data packets will be lost at the controller side. Similarly, Yang *et al.* (2006) also considered the same packet loss approach while modelling the random time delays. Hespanha *et al.* (2007) used Bernoulli's probability distribution function to derive the mathematical model of single packet loss as well as multiple packet loss. In both the cases the situation of packet loss was considered when the network delay is greater than sampling interval. Gupta *et al.* (2007) designed optimal LQG controller that compensates the effect of packet loss occurring within the network. Similarly, Wu and Chen (2007) used the concept of the state estimation to compensate the effect of packet loss in discrete-time domain in NCS. Shi and Yu (2009) assumed the packet loss situation while modelling the random time delays using Markov's chain process. Zhu and Yang (2009) designed state feedback controller with multiple-packet transmission. They proposed the model of NCS with multiple packet transmission and given mathematical model of packet dropout in sensor to controller channel and controller to actuator channel. Niu and Ho (2010) designed the compensator using probability function that compensates the effect of packet loss within the network. Dong and Kim (2012) used Dirac delta probability function to derive the mathematical model of packet loss assuming the single packet loss situation. Wang *et al.* (2013) used the concept of polytopic-uncertainty-based data drift to model the packet loss occurring in sensor to controller and controller to actuator.

They designed the H_∞ controller to compensate the effect of random time delay and packet loss. Wen and Gao (2013) proposed H_∞ controller for NCS in multiple packet transmission with random delays. They modelled multiple packet using markov's chain process. Li *et al.* (2014) designed sliding mode predictive controller under multiple packet transmission policy. Khanesar *et al.* (2015) derived the packet loss model using the concept of uniform distributed probability function with single packet loss assumption. Argha *et al.* (2015) included the random packet loss situation while modelling the random time delays using Bernoulli's distribution. Recently, Song *et al.* (2016) have proposed the packet loss model using Markov chain process. The model was validated for a single packet drop as well as successive packet drops. They proposed discrete-time integral sliding mode controller using the proposed model to compensate the effects of packet loss.

1.2.4 Output feedback

The above all literatures, discusses about design of controllers based on the state information method. The major disadvantage of these controllers is that, its performance depends upon the availability of state information. In various applications of NCS such as missile guidance control, aircraft control, chemical industries and automobile sectors it is not mandatory that all the states information is available. In such cases, it is better to design the controller based on output feedback method. The main advantage of this method is that, the performance of controller depends on the availability of output information which is always available. Recently, many researchers have paid much attention on designing the controllers based on output feedback approach in NCS. Mu *et al.* (2004) proposed Luenberger output feedback based controller for discrete-time networked systems. The controller consists of two parts: a state observer that estimates plant states from the output when it is available via network, and a model of the plant that is used to generate the control signal when plant output is not available. Similarly, Hespanha and Naghshtabrizi (2005) designed observer based Luenberger output feedback to deal with these problems for anticipative and non anticipative control unit in continuous-time domain. Shi and Yu (2009) proved the stability of NCS with random time delays using output feedback method. Zhang and Xia (2011) also designed predictive controller that compensates the effect random delays in the presence of matched uncertainty using output feedback approach. Yu and Antsaklis (2011) introduced the concept of event triggered for designing output feedback controller in NCS in the presence of time varying network delays. Zhang *et al.* (2013*a*) designed output feedback sliding mode controller to study the effect of variable time delay in the presence of matched uncertainty. Jungers *et al.* (2013) proved stability of NCSs including global time varying networked delay. They designed controller based on dynamic output feedback approach dependent on estimation of time varying delay. Similarly, Wang *et al.* (2013) designed output feedback H_∞ controller for NCSs with packet dropouts, network induced delays and data drift. They introduced polytopic uncertainty based data drift to model closed loop NCSs which include random time delay and packet loss. Qiu *et al.* (2013) designed robust output feedback controller for T-S fuzzy based affine systems with unreilable communication links with multiple packets-dropout. Later, Hong *et al.* (2014) designed conventional observer using output feedback approach for wireless NCSs with time varying network delays and packet dropouts. They modelled wireless NCS using asynchronous dynamic system with assumption that time varying network delays can be more or less than sampling interval. Yu *et al.* (2014) designed multiple dynamic output feedback controllers for networked control systems in the presence of random time delays and packet loss.

It is worth to mention here that, the time required for the data packets to travel from sensor to controller and controller to actuator is defined as total network delay. When such delay is transformed into discrete-time domain it mostly possesses non-integer type of values. Such network delays in discrete-time domain are defined as fractional delays. The networked control system has sensor to controller fractional delay present in the feedback channel and controller to actuator fractional delay present in the forward channel. The nature of both these fractional delays depends on the type of the communication medium. In NCS, when the data packets are exchanged through real time communication medium the network delay always have the fractional delay. So, it is important to compensate the effect of deterministic and random type of fractional delay in discrete-time domain at each sampling instant in the presence of packet loss and matched uncertainty.

1.3 Thesis contribution

This thesis contributes mainly following:

- Firstly, a novel discrete-time sliding surface is proposed using the compensated state information and proposed a design of discrete-time sliding mode controller that encompasses deterministic type fractional delay and single packet loss. The Thiran's approximation technique is used for compensating the deterministic type of fractional delays. Two types of Discrete-time Sliding Mode Controllers are proposed that is Switching type and Non-switching type DSMC. The conditions for stability of the closed loop system using proposed controller are derived using the Lyapunov approach. The algorithms are checked with simulation also validated by the experimental results on servo system for various performance parameters. The proposed algorithms are also compared with conventional sliding mode controller using CAN and Switched Ethernet as network medium. The robustness properties of the algorithm are also checked with slowly varying matched uncertainties.

- The above algorithms are using the state information for the controller design but in most of the control scenario only the output information is available. The thesis incorporates the multi-rate output feedback approach for the state estimation in the closed loop. The main advantage of using multi-rate output feedback is that the error between estimated state variables and actual state variables goes to zero once the output sample is available. The proposed multi-rate output feedback discrete-time sliding mode controller also performs well in the networked environment.

- Next, the thesis proposes the discrete-time sliding surface design for the random fractional delay and single packet loss that occur within the sampling period. The random delay is compensated using Thiran's approximation technique in the presence of packet loss situation. The random fractional delay is modelled by Poisson's distribution function and Packet loss are modelled by Bernoulli's function. The closed loop stability is proved using the Lyapunov function. The efficacy of proposed novel non-switching type of DSMC is endowed by simulation results and also experimentally validated by servo system.

- Further, the proposed algorithms extended for the random fractional delay with multiple packet loss situation. The simulation as well experimental results with various fractional delay situation and matched uncertainties show the efficacy of the proposed algorithms.

1.4 Thesis structure

The thesis is structured as follows:

- $\mathbf{Chapter-1}$ briefs about introduction and literature survey for NCS. The chapter discusses a brief introduction of NCS with conceptual model and different stuctures of NCS. Various issues related to NCS are also discussed.

- $\mathbf{Chapter - 2}$ discusses the Preliminaries of Networked Control System and Sliding Mode Control. In this chapter a basic block diagram of NCS with different types of time delays that affect the performance of the system are discussed. The origin of sliding mode controller in continuous-time domain and discrete-time domain are also briefly discussed. Lastly, various challenges of NCS with SMC are also highlighted.

- The main contribution of thesis design of discrete-time sliding mode control for deterministic type fractional delay is mentioned in $\mathbf{Chapter - 3}$. In this chapter, the compensation of determinitic fractional delay is studied through Thiran's Approximation in the sliding surface. The discrete-time sliding mode control law is derived using proposed sliding surface with switching type reaching law. Further, the stability of the closed loop NCS is proved through Lyapunov approach. The efficacy of the proposed algorithm is tested under simulation enviornment and experimental enviornment.

- $\mathbf{Chapter - 4}$ briefs about the designing of non-switching type discrete-time sliding mode controller in the presencce of deterministic fractional delay and matched uncertainty. In this chapter, the design of control law is based on sliding surface derived using Thiran Approximation. Further, the stability of the closed loop NCS is proved through Lyapunov approach that ensures the finite time convergence of system states within the specified band. The efficacy of the proposed algorithm is tested under simulation enviornment, experimental enviornment and real-time networks.

- $\mathbf{Chapter - 5}$ describes the design of discrete-time sliding mode control using multirate output feedback approach with fractional delay compensation. In this chapter, the control signal is computed based on the output measurements available at the controller side and the fractional delay is compensated using Thiran approximation. The stability of the closed loop NCS with derived control law is proved using Lyapunov approach. The simulation results are carried out in the presence of network delay and matched uncertainty in order to prove the effectiveness of proposed algorithm.

- $\mathbf{Chapter - 6}$ describes the design discrete-time sliding mode controller for random communication delay and packet loss. In this chapter, the compensation of random fractional delay is studied using Thiran's Approximation with packet loss condition. The mathematical models of random fractional delay and single packet loss are derived using stochastic approach. The derived discrete-time sliding mode control law is verified through simulation and implementation results in the presence of random fractional delay, packet loss and matched uncertainty.

- $\mathbf{Chapter - 7}$ discusses the mathematical model of multiple packet loss and design of discrete-time sliding mode control for multiple packet transmission. In this chapter, the discrete-time sliding mode control law is designed in the presence of multiple packets transmission. The multiple packet loss is modelled using probability function. The efficiency of the proposed algorithm is verified through simulaton and experimental results.

- The concluding remarks along with future scope and challenges are mentioned in $\mathbf{Chapter - 8}$. In this chapter, final comments and future scope of discrete-time

SMC algorithms are discussed. Lastly, various challenges are also listed that are still remain unsloved in network control system domain.

CHAPTER 2

PRELIMINARIES OF NETWORK CONTROL SYSTEM AND SLIDING MODE CONTROL

2.1 Introduction

In this chapter, we introduce the concept of Networked Control System (NCS), the irregularities such as time delay and packet loss that occurs in the NCS. We also discussed the various control methods available for NCS. This chapter also presents the concept of sliding mode control alongwith literature survey on SMC for NCS.

2.2 Networked Control System (NCS)

NCS is mainly classified in three structures: (i) shared network structure, (ii) hierarchical network structure and (iii) direct network structure. Figure (2.1) shows the block diagram of typical network control system with direct network structure. In this structure, the sensor, plant and actuator are connected to controller through communication network. The communication network in NCS transfers the data in the form of packets. The thick lines indicates the continuous-time data signal while the dotted lines indicates discrete-time data signals. As shown in Figure (2.1) the data signal transmitted from sensor to controller through the network is called as feedback channel while the control signal transmitted from controller to actuator through the same network is called as forward channel. In NCS most of the applications are based on time-sensitive parameter. So, in such cases, if the network delay increases beyond its tolerance limit the plant or the device can either be damaged or gives inferior performance.

2.2.1 Delays and Packet Loss in NCS

In NCS there are four types of time delay (i) sensor to controller delay, (ii) controller to actuator delay, (iii) computational delay and (iv) processing delay. The sensor to con-

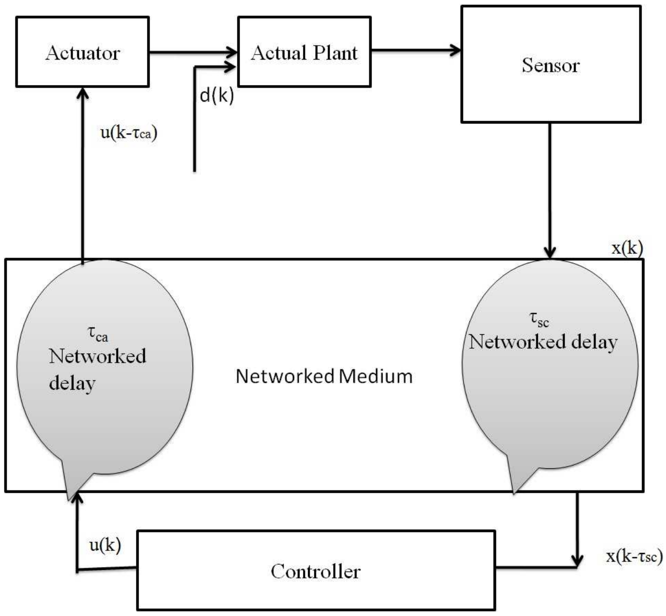


Figure 2.1: Block Diagram of Network Control System

troller delay is present in the forward channel and controller to actuator delay present in the feedback channel. The time required for the data to travel from sensor to controller is called a sensor to controller network delay. And similarly the time required for control signal to travel from controller to actuator is called a controller to actuator delay. The combination of both these delays are defined as network delays. The computational delay and processing delay are caused due to the presence of sensor, controller and actuator. So they are also defined as the system delays.

The combination of system delays and network delays is defined as total delay and mathematically it is represented as:

$$\tau_t = \tau + \tau_p, \tag{2.1}$$

where, τ is the total network delay and τ_p is the system delays.

The mathematical representation of total network delay and system delay is given as:

$$\tau = \tau_{sc} + \tau_{ca}, \tag{2.2}$$

$$\tau_p = \tau_{sp} + \tau_{cp} + \tau_{ap}, \tag{2.3}$$

where, τ_{sc} is sensor to controller delay, τ_{ca} is controller to actuator delay, τ_{ap} is actuator processing delay, τ_{cp} is controller computational delay and τ_{sp} is sensor processing delay.

In NCS it is always assumed that the effect of system delays (τ_p) are negligible compared to network delays (τ). So, Eqn. (2.1) can be written as,

$$\tau_t = \tau. \tag{2.4}$$

Thus, from above Eqn. (2.4), it can be noticed that when real time networks are not considered in NCS the total network delays are always equal to total delay. These network delays can either be deterministic or random in nature.

The nature of the network delays (τ) depends upon the configuration of networks while the system delays (τ_p) are always deterministic in nature. When the communication takes place using the concept of lease line then network delays (τ) are deterministic in nature. And when the communication medium is shared by large number of devices then network delays (τ) are random in nature.

A natural assumption on network delays (τ) can be made as,

$$\tau \prec h, \tag{2.5}$$

Or

$$\tau_l \leq \tau \leq \tau_u. \tag{2.6}$$

where, h is sampling interval, τ_l is lower bound of delay and τ_u is upper bound of delay. Observing condition (2.5) and (2.6), it can be concluded that the network delay should always be bounded. During transmission if the packet gets delayed or fails to arrive at the destination within the specified condition (2.5) and (2.6), then such packets are considered as lost packets within the network.

In NCS, the packet loss occurs either in the forward channel or feedback channel. The packet loss are broadly classify in two different categories based on the distance of communication: (i) If the distance of commmunication is shorter, the data transfer in NCS takes place in the form of frames. Such frame when lost during transmission, is treated as single packet loss. (ii) If the distance of communication is longer, the same frame is breakdown in the form of small packets. Such packets when lost during transmission are defined as multiple packet loss.

2.2.2 Sliding Mode Control with NCS

Many researchers have proposed different controller design methodologies in discrete-time as well continuous-time domain for NCSs such as Gain Scheduler Middleware [Tipsuwan and Chow (2004)], H_∞ [Yue *et al.* (2005)], State Feedback [[Peng and Yue (2006)] [Gao *et al.* (2007)], Sliding Mode Controller (SMC) [Mehta and Bandyopadhyay (2009)], Adaptive Controller [Vallabhan *et al.* (2012)], Smith Predictive [Hikichi *et al.* (2013)], Back-stepping Controller [Yi *et al.* (2014)], fuzzy-based sliding mode control [Khanesar *et al.* (2015)] etc... Among all these controllers SMC has been an active research area in the field of NCS because it has ability to reject the disturbances which makes it more robust.

In past few decades many researchers have tried to implement the SMC algorithm in various ways to compensate the effects of network delay and packet loss in NCSs. Wang *et al.* (2011) proposed variable structure control algorithm for time delay system. They proposed the controller based on the concept of smith predictor. Gao (2013) designed integral type of sliding mode control in continuous-time domain in the presence of variable time delay and matched uncertainty. They proposed sliding mode compensator in reaching law that compensates the effect of variable network delay. Goyal *et al.* (2015) designed fuzzy-based sliding mode control in continuous-time domain. They considered the state-based delay rather than control input delay. Recently, Khanesar *et al.* (2015) proposed indirect fuzzy based sliding mode control in continuous-time domain for NCS. The effect of random time delay was compensated using Pade approximation and packet loss was compensated using probability distribution function.

With the rapid development in digital controllers, various researchers have contributed their work in discrete-time domain. The main advantage of designing the controllers in discrete-time domain is that the effects of control signal can be observed very clearly at each sampling instant. Moreover, in case of NCS since the communication is carried out in digital signal form so it better to design SMC in discrete-time rather than continuous-time domain. Xiaojuan and Jinglin (2010) designed fuzzy based sliding mode control for NCS and studied the effect of time delay using Ethernet as a network medium in discrete-time domain. Niu and Ho (2010) designed sliding mode control that compensates the effect of single packet loss using probability distribution function. Yin *et al.* (2011) designed adaptive based sliding mode control in the presence of variable time delay and uncertainties. They considered sensor to controller delay and transformed the control signal into non-delayed form using fuzzy fusion system in discrete-time domain. Zhang *et al.* (2013*a*) designed output feedback based sliding mode controller to study the effect of variable time delay in the presence of matched uncertainty. Li *et al.* (2014) designed sliding mode controller using Kalman predictor in the presence of multiple packet transmission. The Kalman predictor was used to estimate the integer type of network delays. Goyal *et al.* (2015) proposed robust sliding mode control for nonlinear discrete-time delayed system based on neural network. They assumed network delay as deterministic in nature. Argha *et al.* (2015) proposed discrete-time sliding mode control that compensates the effect of random time delay and packet loss using Bernoulli's distribution. They considered integer type of network delay in discrete-time domain. Recently, Song *et al.* (2016) designed integral sliding mode controller that compensates the effect of single as well as multiple packet loss using markov's chain process.

2.3 Brief Review of Sliding Mode Control (SMC) Technique

2.3.1 Origin of SMC

The concept of Variable Structure Control (VSC) was introduced by Emelyanov group in late 1950's (Emelyanov and Korovin, 1981). The main idea of VSC was to switch between the various control structures according to the evaluation of the system states. The concept of VSC can be understand through the following example.
Consider two constituent systems given as:

$$\ddot{x} = -a_1 x, \tag{2.7}$$

$$\ddot{x} = -a_2 x, \tag{2.8}$$

where, $0 \prec a_2 \prec a_1$. The phase potraits of the systems are shown in Figure (2.2) and (2.3) repectively. It can be observed that both the systems are oscillartory in nature and are unstable. But, when both structures are switched at appropriate time, then, combined system is asymptotically stable. The combined response of both the system is shown in Figure (2.4). Thus it can be noticed, that the property not present in any of the system is obtained by VSC.

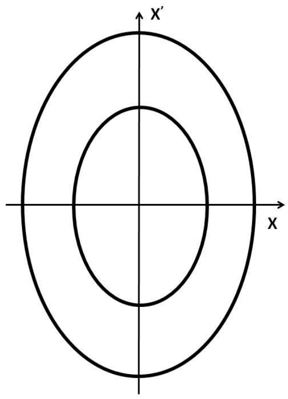


Figure 2.2: State Trajectories of system in Mode-I

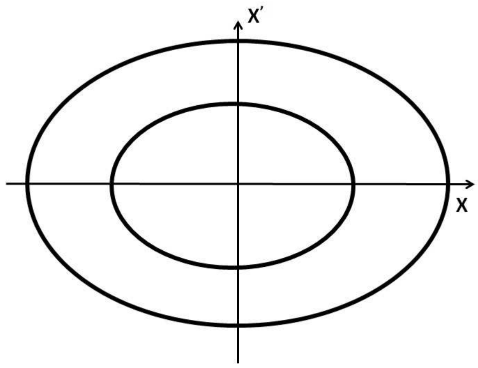


Figure 2.3: State Trajectories of system in Mode-II

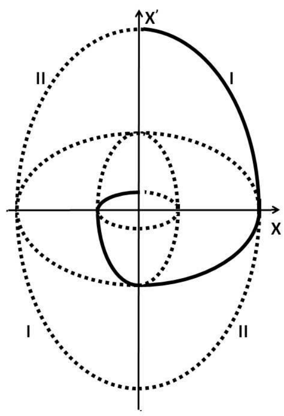


Figure 2.4: Combined system response

While carrying out research on VSC some of the researchers [Utkin (1993), Edwards and Spurgeon (1996), Drakunov *et al.* (1990), Furuta (1990)] made an unusual observation that switching between two or more unstable control structures may result in stable control system. They introduced the notions of variable structure control, variable structure system and a new control idea called sliding mode control came into existence. The sliding mode control, or SMC, is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behaviour. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move towards an adjacent region with a different control structure, and so the ultimate trajectory will not exist entirely within one control structure. Instead, it will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding surface. The conventional example of sliding mode is second order relay system which is given by,

$$\ddot{x} + a_2\dot{x} + a_1 x = u, \tag{2.9}$$

$$u = -M_s sign(s), \tag{2.10}$$

$$s = cx + \dot{x}, \tag{2.11}$$

where, a_1, a_2, M_s, c are contants. From Eqn. (2.10) it can be noticed that the control input in the second order system might take only two values M_s and $-M_s$ and causes discontinuities on the straight line $s = 0$ in the state plane $(x, \dot{x})$. Figure (2.5) shows the response of the sliding mode for the specified example with $a_1 = a_2 = 0$. From the result it can be observed that in the neighbourhood segment mn on the switching line $s = 0$, the trajectories of the system (2.9) runs in the opposite directions which leads to the appearance in sliding mode along this line [Sabanovic *et al.* (2004)]. Thus the Eqn. (2.11) can be interpreted as sliding mode equation. Further, it can be noticed that the order of sliding mode equation is less than the original system (2.9). Thus it can be said that the sliding mode does not depend on plant dynamics but it is determined by parameter c only. It is worth to point out that for desired performance of the closed loop system not only the sliding mode controllers have to be properly designed but the switching rule should also be chosen properly. Based on the appropriate selection of switching rule and designed controller the system states will drive onto the predefined sliding surface in finite time. This ensures stability of sliding motion on the surface and desired dynamic characteristics of the systems are achieved.

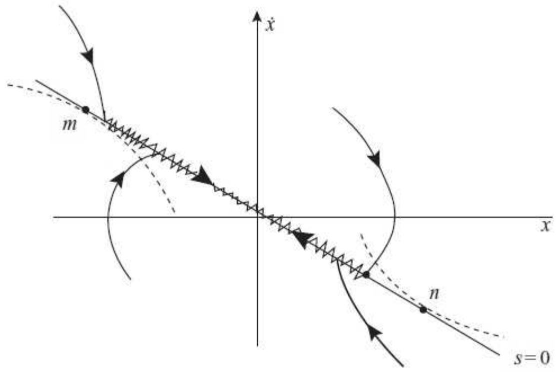


Figure 2.5: Sliding mode for relay system

Due to its lower sensitivity towards plant parameters variations and external disturbances it eliminates the necessity of exact modelling. It allows the decoupling of overall system motion into partial components of lower dimensions. This reduces the complexity of feedback design. In sliding mode control the control actions are function of discontinuous state which can be easily implemented using conventional power converters. Due to such properties SMC has been proved applicable to a wide range of applications such as robotics, electrical drives, electrical generators, motion control, process control and networked control system.

2.3.2 Continuous-time Sliding Mode Control

In sliding mode control the control law consists of two important phases: (i) reaching phase (ii) sliding mode phase. When the system state is driven from any initial state to reach the switching manifold in finite time then such phase is called reaching phase and when the system is induced into the sliding motion on the switching manifolds, then such phase is defined as sliding mode phase. Figure (2.6) shows the phases of sliding mode control with s as continuous-time sliding function given by:

$$s = \{x \in X | s(x,t) = 0\}. \tag{2.12}$$

In order to induce the sliding mode following properties should exist: (i) the system

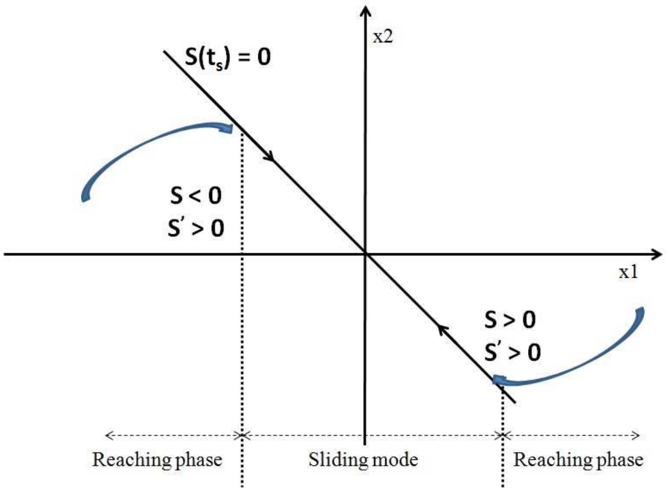


Figure 2.6: Phases of Sliding Mode Control

stability should be restricted to the sliding surface and (ii) the sliding mode should start within a finite time. The sufficient condition for the sliding motion to slide on the given surface is given by:

$$s\dot{s} \prec 0, \tag{2.13}$$

where, s is the sliding surface and $\dot{s}$ is rate of change of distance from the sliding surface. The condition specified in Eqn. (2.13) is called a reachability condition. The reachability condition is not sufficient for the sliding mode. The main drawback of condition mentioned in (2.13) is that, $s(t)$ takes infinite time to reach on the sliding surface. Thus, to over come this drawback another condition is defined as:

$$s\dot{s} \prec -\eta|s|, \eta \succ 0. \tag{2.14}$$

This condition is known as 'η-reachability' condition that ensures the finite time convergence to $s = 0$.

As discussed earlier, the designing of the sliding mode controller includes reaching law design, sliding surface design and control law design. Let us consider the continuous-time system given by:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{2.15}$$

$$y(t) = Cx(t). \tag{2.16}$$

The dynamics of the sliding function can be expressed in the form of constant-rate reaching law as:

$$\dot{s} = -k_s sgn(s), k_s \succ 0. \tag{2.17}$$

Let, the sliding surface be given by:

$$s(t) = C_s x(t), \tag{2.18}$$

where, C_s is the sliding gain that can be computed using pole-placement method based on the proper slection of poles or LQR method based on the proper slection of Q and R matrices.

Thus, the control law for the system (2.15, 2.16) is derived by using the condition $\dot{s} = 0$ as:

$$u(t) = -(C_s B)^{-1}[AC_s x(t) + k_s sgn(s)]. \tag{2.19}$$

The reaching laws proposed in the literatures [Fallaha *et al.* (2011), Mehta and Bandyopadhyay (2015)] are,

- Constant-proportional rate

$$\dot{s} = -qs - k_s sgn(s), q \succ 0 \tag{2.20}$$

- Power-rate reaching law

$$\dot{s} = -k_s |s|^{\iota} sgn(s), 0 \prec \iota \prec 1 \tag{2.21}$$

- Exponential reaching law

$$\dot{s} = -\frac{k_s}{N(s)} sgn(s), \tag{2.22}$$

where, $N(s) = \delta_0 + (1 - \delta_0)e^{-\iota|s|^{p_0}}$, δ_0 is strictly positive offset less than 1, p_0 is a strictly positive integer.

The main limitation of continuous-time SMC is that once the closed loop system states reach on the sliding surface, a discontinuous control term switches with high frequency which results in chattering phenomenon. The chattering is caused due to various reasons such as switching time delay, controller computation delay, dynamics of plant elements such as actuator and sensor etc... In practical applications this phenomenon is not desirable as it affects the performance of plant. While in electrical and mechanical applications it causes high heat losses and generates wear and tear of the moving parts of machines. In discrete-time sliding mode control generally low switching frequencies are required because of limited sampling frequency due to which it becomes more useful for practical implementation. Moreover, in discrete-time SMC the computation of control signal is done at each sampling interval and remains constant for that period. So due to this the system state trajectory is unable to move along the sliding surface but follows the zigzag motion about the sliding surface defined as quasi-sliding mode motion.

2.3.3 Discrete-time Sliding Mode Control

The concept of discrete-time sliding mode control is first introduced by Milosavljevic (1985). In his work he proved that the sliding motion in discrete-time is more accurate than continuous-time sliding mode control. Later on, Sarpturk *et al.* (1987), Utkin (1993), Bartolini *et al.* (1995), Gao *et al.* (1995), Bartoszewicz (1996, 1998), Su *et al.* (2000) and many others extended their work in discrete-time sliding mode control.

The designed procedure of DSMC includes: (i) design of sliding surface, (ii) reaching law and (iii) control law that steers the system states to slide along predefined sliding surface over a finite interval of time. In discrete-time SMC Gao's introduced the concept of switching function in the reaching law that causes the system states to move towards the vicinity of the origin but cannot get arbitrarily closed to the origin. While Bartolini and Bartoszewicz designed the reaching law without considering the switching function. They considered that discrete-time control is naturally discontinuous in nature and thus may not require an explicit discontinuity in the control law. The reaching law proposed by them causes the system states to get arbitrarily closed to the origin. Various state-based discrete-time control algorithms are designed using different reaching laws available in Milosavljevic (1985), Sarpturk *et al.* (1987), Gao *et al.* (1995), Bartolini *et al.* (1995) and Bartoszewicz (1996, 1998) respectively.

In order to derive the discrete-time control algorithms, let us consider the continuous-time SISO system as:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{2.23}$$

$$y(t) = Cx(t), \tag{2.24}$$

where, $x \in R^n$ respresents system state vector, $u \in R^m$ represents control input, $y \in R^p$ represents system output, $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{p\times n}$ are the matrices of appropriate dimensions.

Let the system (2.23) and (2.24) be discretized at h sampling interval given by,

$$x(k+1) = Fx(k) + Gu(k), \tag{2.25}$$

$$y(k) = C(k). \tag{2.26}$$

As discussed earlier, the designed of sliding mode control algorithms involves the design of sliding surface and reaching law. Let the discrete-time sliding surface be given by:

$$s(k) = C_s x(k). \tag{2.27}$$

Various researchers have proposed famous reaching laws in discrete-time domain as listed below.

- Sarpturk's Reaching Law: The reaching law proposed by Sarpturk *et al.* (1987) is the direct discretization of continuous-time sliding mode given by:

$$|s(k+1)| \prec |s(k)|. \tag{2.28}$$

  Here, the sliding surface is always directed towards the surface and also the norm of $|s(k)|$ monotonically decreases. The reaching law can be written in other way as,

$$(s(k+1) - s(k))sgn(s(k)) \prec 0, \tag{2.29}$$
$$(s(k+1) - s(k))sgn(s(k)) \succ 0. \tag{2.30}$$

  The first condition (2.29) indicates that the closed loop system state trajectories should move towards the direction of sliding surface and the second condition (2.30) indicates that the closed loop system state trajectories are not allowed to go too far in that direction. Thus observing condition (2.29) and (2.30), will lead to lower and upper bounds for control actions. The control law proposed in [Sarpturk *et al.* (1987)] is given as,

$$u(k) = -k_t(x, s)x(k), \tag{2.31}$$

  where, k_t is the gain given by:

$$k_t(x, s) = \begin{cases} k_t^+; & \text{when } x(k)s(k) \succ 0 \\ k_t^-; & \text{when } x(k)s(k) \prec 0 \end{cases}$$

  where, k_t^+ and k_t^- represents the coefficients of each upper bound and lower bound of control action that can be determined by evaluating the condition (2.29) and (2.30) respectively.

- Gao's Reaching Law: The switching based reaching law proposed by Gao *et al.* (1995) is given as,

$$s(k+1) = (1-qh)s(k) - \epsilon h sgn(s(k)), \tag{2.32}$$

where, h is the sampling interval satisfying $h \succ 0$, $q, \epsilon \succ 0$ and $1 - qh \succ 0$.
Using reaching law (2.32), the switching based control law for system (2.25, 2.26) is computed as,

$$u(k) = -(C_s G)^{-1}[F C_s x(k) - (1-qh)s(k) + \epsilon h sgn(s(k))]. \tag{2.33}$$

From above Eqn. (2.33), it can be noticed that there are two parameters q and ϵ in control law for tuning the response. The discrete-time sliding mode control law proposed in (2.33) should achieve the following performances [Gao *et al.* (1995)]. (i) Starting from any initial state, the trajectory will move monotonically toward the switching plane and cross it in finite time. (ii) Once the trajectory has crossed the switching plane, it will cross the plane again in every successive sampling period, resulting in a zigzag motion about the switching plane. and (iii) The size of each successive zigzag step is nonincreasing and the trajectory stays within a specified band.
The reaching law in Eqn. (2.32) states that the state vector always move towards the quasi-sliding mode band given as:

$$s(k) \leq \frac{\epsilon h}{1-qh}. \tag{2.34}$$

From Eqn. (2.34), it can be observed that ϵ is directly proportional to quasi-sliding mode band. Thus if the value of ϵ is too large than the system will have high overshoots and could also increase the transient response. While on the other hand the value of qh should be less than unity otherwise it will speed up the transient response.

- Bartoszewicz's Reaching Law: Bartoszewicz (1996, 1998) proposed non-switching reaching law as,

$$s(k+1) = d(k) - d_0 + s_d(k+1), \tag{2.35}$$

where, $d(k)$ is the unknown disturbance, d_0 is the mean value of disturbance $d(k)$ and μ_d is minimum deviated disturbance with d_u as upper bound and d_l as lower bound. Also, d_0 and d_2 is given by

$$d_0 = \tfrac{d_l + d_u}{2} \text{ and } d_2 = \tfrac{d_u - d_l}{2}$$

$s_d(k)$ is an priori known function such that the following applies:

- If $s(0) \succ 2d_2$ then,

$$\begin{gathered} s_d(0) = s(0) \\ s_d(k)s_d \geq 0 \text{ for any } k \geq 0 \\ s_d(k) \geq 0 \text{ for any } k \geq k' \\ |s_d(k+1)| \prec |s_d(k)| - 2d_2 \text{ for any } k \leq k' \end{gathered}$$

These relations state that $s_d(k)$ converges monotonically from its initial position to the origin of the state space in a finite time. Moreover, in each control step the hyperplane moves by the distance greater than $2d_2$. This, together with Eqn. (2.35), states that even in the case of worst combination of disturbance the reaching condition is satisfied.

- If $s(0) \prec 2d_2$ then $s_d(k) = 0$ for any $k \geq 0$.
  The k' in the above relations is a positive integer user defined constant which provides the faster convergence rate of the system and magnitude of the control signal $u(k)$. The control law computed using the reaching law (2.35) for the system (2.25, 2.26) is given as,

$$u(k) = (C_s G)^{-1}[C_s F x(k) + d_0 - s_d(k+1)]. \tag{2.36}$$

  The reaching law in Eqn. (2.35) states that the system state vector always move towards the QSMB for any $k \geq k'$ such that:

$$|s(k)| = |d(k-1) - d_0| \leq d_2. \tag{2.37}$$

- Bartoszewicz's Reaching Law: Bartoszewicz and Lesniewski (2014) proposed the other reaching law that provides the faster convergence of sliding variable without increasing the amplitude of the control signal. The reaching law is given as,

$$s[(k+1)h] = \{1 - q[s(kh)]\}s(kh) - \hat{S}(kh) - d(kh) + d_1 + S_1, \tag{2.38}$$

  where, $\hat{S}(kh)$ represents the model uncertainty on sliding variable evolution and $d(kh)$ represents the effect of disturbance on this variable. Further, S_1 and d_1 represents the mean values of $\hat{S}(kh)$ and $d(kh)$ repectively given as,

$$S_1 = \frac{S_u + S_l}{2}, \tag{2.39}$$

$$d_1 = \frac{d_u + d_l}{2}, \tag{2.40}$$

  where, S_u, S_l are upper and lower bounds of S_1 and d_u, d_l are upper and lower bounds of d_1.
  The convergence rate factor of $q[s(kh)]$ in Eqn. (2.38) is given as,

$$q[s(kh)] = \frac{\psi}{\psi + |s(kh)|}, \tag{2.41}$$

  where, ψ is designer's constant satisfying $\psi \succ S_2 + d_2$, where S_2 and d_2 the greatest possible deviation of $\hat{S}$ and d. They are represented as,

$$S_2 = \frac{S_u - S_l}{2}, \tag{2.42}$$

$$d_2 = \frac{d_u - d_l}{2}. \tag{2.43}$$

- The control law computed the reaching law (2.38) for the system (2.25, 2.26) is given as,

$$u(k) = (C_sG)^{-1}[C_sFx(k) + \{1 - q[s(kh)]\}s(kh) + S_1 + d_1]. \tag{2.44}$$

The reaching law in Eqn. (2.38) states that the system states always move towards the QSMB for any $k \succ k_0$ such that:

$$|s(kh)| \leq \frac{\psi(S_2 + d_2)}{\psi - (S_2 + d_2)}. \tag{2.45}$$

2.3.4 Advantages of discrete-time sliding mode control over continuous-time sliding mode control

- With the increase in use of digital computers and microcontrollers for the implementation of control algorithms, a discrete-time model of the system is justified.
- To validate the better performance of the system it is better to implement the discrete-time algorithms rather continuous-time algorithms.
- In continuous-time sliding mode control due to high-frequency switching chattering takes place which may cause damage to the system. So, it is not used for all practical applications. While, in discrete-time sliding mode control relatively low switching frequencies are required so DSMC algorithm is more practical to implement.
- When continuous-time algorithms are implemented using digital controllers for implementation, the chattering generated around the sliding mode and stability of the sliding mode are compromised.
- A large class of discrete-time systems are computer controlled and information about the system measurements are available only at specific time instances and control inputs can only be changed at these time instances. Eg. - biological systems, thyristor, radar system, economic systems, etc.

2.4 Challenges in NCS with SMC

There are various challenges in network control system with sliding mode control that has not been explored. Some of them are listed below:

- In literature, the researchers have proposed various SMC algorithms with time delay compensation [1, 8, 12, 20, 30, 31, 34, 41, 44, 63]. But in all the papers, it is assumed that the delays are mutiples of sampling interval. However, when the communication is carried through real-time network the delays always have fractional behaviour. Thus, there is need to design SMC for fractional delay in discrete-time domain.

- Futhermore, it can also be noticed that none of the literatures on SMC discusses about the designing of sliding surface that compensates the effect of fractional delay. So there is a need of designing the sliding surface such that it compensates the effect of fractional delay.

- The compensation of network delays and packet loss are done at the controller side. None of the researcher has tried to compensate their effects in the sliding surface.

- Till now the sliding mode controllers are designed based on the presence of multiple packet transmission but, the compensation of multiple packets loss with sliding mode control in discrete-time domain is still an open research problem in NCSs.

- Various researchers have tried to explore their work on designing the sliding mode controllers based on conventional output feedback method in the presence of network delay and packet loss. But, the compensation algorithm based on output feedback method in the field of NCSs have not been much explored.

- The designing of the higher order sliding modes in the presence of network nonidealities (such as random time delay, packet loss and matched uncertainty) is still an open research area in NCSs.

2.5 Conclusion

In this chapter, we introduced the basic concept of NCS and its configurations. The various irregularities of NCS are also discussed. The Sliding Mode Control (SMC) technique for NCS are also explained and at last challenges for designing the SMC for NCS are also discussed.

CHAPTER 3

DESIGN OF DISCRETE-TIME SLIDING MODE CONTROLLER (SWITCHING TYPE) FOR FRACTIONAL DELAY

3.1 Introduction

In this chapter, a novel approach for designing a discrete-time sliding mode controller using Thiran's delay approximation is presented, to compensate the effect of sensor to controller fractional delay and controller to actuator fractional delay in discrete-time domain. The forward channel delay is compensated at the actuator side while feedback channel delay is compensated at the sliding surface. A novel sliding surface is proposed using delay compensation and a SMC law is derived. The stability condition for the closed loop system with proposed controller is derived using Lyapunov function. The efficacy of the proposed algorithm is shown by simulation results and also validated by the experimental results considering DC Servo motor as plant in the presence of deterministic network delays and matched uncertainty.

3.2 Networked Control System with Fractional Delay Compensation

Figure (3.1) potrays the block diagram of NCS with time delay compensation scheme. It can be noticed that the state information as well as control information are transmitted to the controller and actuator through the network. During transmission, the state information will experience sensor to controller delay while the control information will suffer from controller to actuator delay. These delays are broadly defined as the amount of time required for the data packets to travel within the network. Thus, in order to avoid the degradation it is necessary to compensate these network delays at the controller end as well as at actuator end. Moreover apart from these network delays it is necessary to consider the system delays.

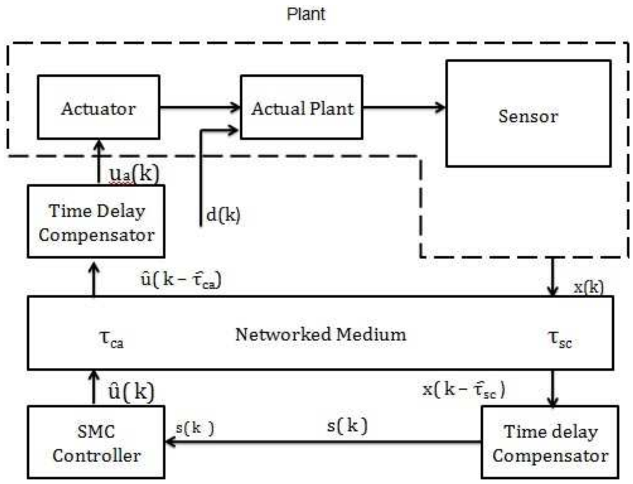


Figure 3.1: Block diagram of NCS with time delay compensation

3.3 Problem Formulation

Consider the linear time invariant SISO system with network delay as:

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dd(t), \quad (3.1)$$

$$y(t) = Cx(t), \quad (3.2)$$

where $x \in R^n$ is system state vector, $u \in R^m$ is control input, $y \in R^p$ is system output, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{n\times m}$ are the matrices of appropriate dimensions, $d(t)$ is the matched bounded disturbance with $|d(t)| \leq d_{max}$ and τ is the total networked induced delay in continuous-time domain.

The discrete form of system (3.1) and (3.2) is:

$$x(k+1) = Fx(k) + Gu(k-\tau') + d(k), \quad (3.3)$$

$$y(k) = Cx(k), \quad (3.4)$$

where $F = e^{Ah}$, $G = \int_0^h e^{At}Bdt$, $d(k) = \int_0^h e^{At}Dd((k+1)h-t)dt \in O(h)$. Since $|d(t)| \leq d_{max}$, it can be inferred that $d(k)$ is also bounded and $O(h)$ [Mehta and Bandyopadhyay (2016)]. For simplicity, it is assumed that $d(k)$ is slowly varying and remain constant over the interval $kh \leq t \leq (k+1)h$ [Mehta and Bandyopadhyay (2016)]. The network induced fractional delay (τ') occurring within the network denoted as,

$$\tau' = \frac{\tau}{h},$$

where h is the sampling interval and τ is the total networked induced delay.

$\mathbf{Remark - 1}$: In this work, it is considered that network induced fractional delay (τ') in discrete-time domain have non-interger values so that the precise effect of network delays are compensated at each sampling instants.

$\mathbf{Assumption - 1}$: The total network induced fractional delay is deterministic in nature satisfying,

$$\tau \prec h, \tag{3.5}$$

where τ indicates the total network delay in continuous-time domain.

$\mathbf{Remark - 2}$: The above condition (3.5) indicates that the values of total network induced fractional delay (τ') in discrete-time domain will be less than unity.

The total network induced fractional delay is the combination of sensor to controller fractional delay (τ'_{sc}) and controller to actuator fractional delay (τ'_{ca}) which is given as,

$$\tau' = \tau'_{sc} + \tau'_{ca}, \tag{3.6}$$

where, $\tau'_{sc} = \frac{\tau_{sc}}{h}$ and $\tau'_{ca} = \frac{\tau_{ca}}{h}$.

$\mathbf{Assumption - 2}$: The disturbance $d(k)$ is bounded by upper and lower bound as:

$$d_l \leq d(k) \leq d_u, \tag{3.7}$$

where, d_l and d_u denotes lower and upper bound of disturbance.

$\mathbf{Remark - 3}$: In this work, the sensor processing delay τ_{sp}, controller computational delay τ_{cp} and actuator processing delay τ_{ap} are neglected as their values are $(\frac{1}{4})^{th}$ times lesser than network induced delay.

$\mathbf{Problem\ Statement}$: To design robust discrete-time sliding mode controller for the system (3.3,3.4) in the presence of deterministic fractional network delays τ'_{sc} and τ'_{ca} under the assumptions (1) and (2).

The sliding mode controller design involves the sliding surface design and the control law that computes the control sequences and steer the states towards the surface. The next segment proposes the design of sliding surface that compensates the effect of fractional delay occuring from sensor to controller.

3.4 Sliding Surface Design for Deterministic Network Delays

Tustin approximation and Bilinear transformation are two widely used approaches for approximation of time delay in discrete-time domain. The main limitation of these two approaches are that fractional delay cannot be approximated. While, Thiran approximation [Thiran (1971)] technique approximates the non-integer types of delays in discrete-time domain. Thiran has proposed the time delay approximation algorithm for maximally flat group of fractional delays occuring in signal processing applications. Hence it is proper candidate for fractional delay compensation for discrete-time SMC design.

The fractional delay in discrete-time can be approximated by Thiran's approximation as:

$$z^{-\nu} = \Sigma_{k=0}^{l}(-1)^k \binom{l}{k} \Pi_{i=0}^{l} \frac{2\tau'_{sc} + i}{2\tau'_{sc} + k + i} z^{-k}, \tag{3.8}$$

where, l indicates the order of approximation and $\nu = \frac{\delta}{h}$ indicates the fractional part of delay, δ is the delay occuring during signal transmission and h is the sampling interval. The order of approximation is given by:

$$l = ceil(\nu), \tag{3.9}$$

where, $ceil$ operator rounds the nearest positive integer greater than or equal to ν.

Next, the sliding surface using above approximation is proposed as $\mathbf{Lemma-1}$

below.

$\mathbf{Lemma-1}$: The compensated sliding variable $s(k)$ for the given system (3.3,3.4) with sensor to controller network fractional delay (τ'_{sc}) satisfying condition (3.5) under the assumptions (1) and (2) is given as:

$$s(k) = C_s x(k) - \alpha C_s(x(k-1)), \tag{3.10}$$

where,
$\alpha = \frac{\tau'_{sc}}{\tau'_{sc}+1}$ and C_s is the sliding gain.

$\mathbf{Proof}$: The sliding variable with the delayed state vector at the receiving end of controller is given by:

$$s(k) = C_s x(k - \tau'_{sc}), \tag{3.11}$$

where, τ'_{sc} is the sensor to controller fractional delay. The sliding gain C_s is calculated using discrete LQR method through proper selection of Q and R matrices.

Applying z-Transform to Eqn. (3.11) we get,

$$s(z) = C_s x(z) z^{-\tau'_{sc}}, \tag{3.12}$$

where, $\tau'_{sc} = \nu = \frac{\tau_{sc}}{h}$.
It is assumed that total fractional delay (τ') is less than unity. Considering $\tau'_{sc} \prec 1$ and using Eqn. (3.9), $z^{-\tau'_{sc}}$ can be approximated as,

$$z^{-\tau'_{sc}} = \Sigma_{k=0}^{1}(-1)^k \binom{l}{k} \Pi_{i=0}^{1} \frac{2\tau'_{sc}+i}{2\tau'_{sc}+k+i} z^{-k}. \tag{3.13}$$

The above Eqn. (3.13) can be further expanded as,

$$\begin{aligned} z^{-\tau'_{sc}} = [(-1)^0 \binom{1}{0} \left\{ \frac{2\tau'_{sc}}{2\tau'_{sc}} * \frac{2\tau'_{sc}+1}{2\tau'_{sc}+1} \right\} z^0 + (-1)^1 \binom{1}{1} \\ \left\{ \frac{2\tau'_{sc}}{2\tau'_{sc}+1} * \frac{2\tau'_{sc}+1}{2\tau'_{sc}+2} \right\} z^{-1}]. \end{aligned} \tag{3.14}$$

On simplifying we get,

$$z^{-\tau'_{sc}} = 1 - \alpha z^{-1}, \tag{3.15}$$

where, $\alpha = \frac{\tau'_{sc}}{\tau'_{sc}+1}$.

Figures (3.2) and (3.3) shows the step response of delayed and non-delayed system for τ'_{sc}=1 and τ'_{sc}=0.5 respectively. It can be noticed that Thiran's Approximation approximates the fractional delay accurately and at each sampling instants the effect of fractional delay is nullified. It can be further extended that the step response of delay system is same as that of the non-delayed system that is computed through Thiran Approximation.

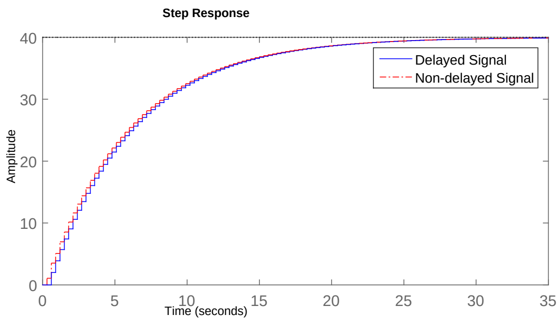


Figure 3.2: Step reponse of Thiran Approximation with τ'_{sc}=1

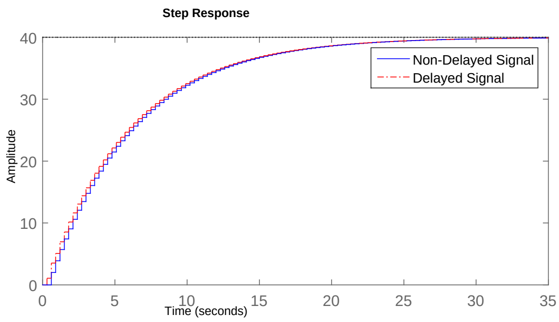


Figure 3.3: Step reponse of Thiran Approximation with τ'_{sc}=0.5

Thus, substituting Eqn. (3.15) into (3.12),

$$s(z) = C_s x(z)[1 - \alpha z^{-1}]. \tag{3.16}$$

Further expanding it, we may get

$$s(z) = C_s x(z) - \alpha C_s z^{-1} x(z). \tag{3.17}$$

Applying inverse z-Transform, we may have

$$s(k) = C_s x(k) - \alpha C_s s x(k-1). \tag{3.18}$$

This completes the $\mathbf{proof}$.

From Eqn. (3.18), it is inferred that the network induced fractional delay from sensor to controller can be compensated in the sliding surface $s(k)$ at each sampling instant h using the information of immediate past state, the current state and parameter α.

Now, we are ready to design a sliding mode control law using the proposed sliding surface (3.18).

3.5 Discrete-Time Sliding Mode Control (Switching type) for NCS Using Thiran's Delay Approximation

In this section, switching type control law alongwith its stability is proposed based on Gao's reaching law [Gao *et al.* (1995)] using compensated sliding surface (3.18). The Gao's reaching law provides the faster convergence within the specified quasi-sliding mode band.

$\mathbf{Theorem-1}$: The discrete-time sliding mode controller for system $(3.3, 3.4)$ in the presence of deterministic fractional delays satisfying (3.5) and matched uncertainty $d(k)$ is given as,

$$u(k) = -(C_sG)^{-1}[Mx(k) - Nx(k) - (1-qh)(s(k)) + \epsilon h sgn(s(k))] - d(k). \tag{3.19}$$

where,

$M = (C_sF)$ and $N = \alpha C_s$.

$\mathbf{Proof}$: Let us consider the reaching law in [Gao *et al.* (1995)] with sensor to controller fractional delay as:

$$s[(k+1)] = (1-qh)s(k) - \epsilon h sgn(s(k)), \tag{3.20}$$

where,

q, $\epsilon \succ 0$, $0 \prec (1-qh) \prec 1$, sgn is the signum function, h represents the sampling interval and $s(k)$ is the compensated sliding surface (3.18).

The reaching law in Eqn. (3.20) states that the state vector always move towards the quasi sliding mode band given as:

$$s(k) \leq \frac{\epsilon h}{2-qh}. \tag{3.21}$$

Substituting Eqn. (3.18) in Eqn. (3.20), we may get:

$$C_s x(k+1) - \alpha C_s x(k) = (1-qh)s(k) - \epsilon h sgn(s(k)).$$

Substituting the value of $x(k+1)$, we get

$$C_s[Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k) = (1-qh)s(k) - \epsilon h sgn(s(k)). \tag{3.22}$$

Further, simplifying we may write above Eqn. (3.22) as,

$$C_s[Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k) = (1-qh)s(k) - \epsilon h sgn(s(k)). \tag{3.23}$$

Further, above Eqn. (3.23) can be expressed as a control law

$$u(k) = -(C_s G)^{-1}[Mx(k) - Nx(k) + (1-qh)(s(k)) - \epsilon h sgn(s(k))] - d(k). \tag{3.24}$$

where,

$M = (C_s F)$ and $N = \alpha C_s$.

This completes the $\mathbf{proof}$.

The stability condition is derived further using compensated sliding surface (3.18) and control law proposed in Eqn. (3.24) such that the system states remain within specified band (3.21) for a finite interval of time.

3.5.1 Stability Analysis

The state trajectories of the closed loop system (3.3,3.4) with network delay (τ') and matched uncertainty $d(k)$ with the controller (3.24) drive towards the sliding surface (3.18) and maintains on it for any $q, \epsilon, \beta \succ 0$, $0 \prec 1 - qh \prec 1$ and $1 - qh \prec \epsilon$ provided the following condition hold true:

$$0 \preceq \Phi \prec s^T(k)s(k). \tag{3.25}$$

Proof: The compensated sliding surface (3.18) is given by:

$$s(k) = C_s x(k) - \alpha C_s x(k-1). \tag{3.26}$$

Selecting the Lyapunov function as:

$$V_s(k) = s^T(k)s(k). \tag{3.27}$$

Writing forward difference of the above Eqn. (3.27),

$$\Delta V_s(k) = s^T(k+1)s(k+1) - s^T(k)s(k). \tag{3.28}$$

Substituting the value of $s(k+1)$ using Eqn. (3.26) we get,

$$\begin{aligned} \Delta V_s(k) = [C_s x(k+1) - \alpha C_s x(k)]^T [C_s x(k+1) \\ -\alpha C_s x(k)] - s^T(k)s(k). \end{aligned} \tag{3.29}$$

Substituting the value of $x(k+1)$,

$$\begin{aligned} \Delta V_s(k) = [C_s[Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k)]^T \\ [C_s Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k)] - s^T(k)s(k). \end{aligned} \tag{3.30}$$

Substituting the value of $u(k)$ from Eqn. (3.24) and further solving it we have,

$$\Delta V_s(k) = [(1-qh)s(k) - \epsilon hsgn(s(k))]^T * [(1-qh) \\ s(k) - \epsilon hsgn(s(k))] - s^T(k)s(k). \tag{3.31}$$

Denoting,

$$\Phi = [(1-qh)s(k) - \epsilon hsgn(s(k))]^T * [(1-qh)s(k) - \epsilon hsgn(s(k))]$$

Then we have,

$$\Delta V_s(k) = \Phi - s^T(k)s(k). \tag{3.32}$$

The term Φ can be tuned close to zero by appropriately selecting the parameter q and ϵ. If Φ is closed to zero, then $s^T(k)s(k)$ will be larger than Φ. Thus, for any small parameter β, we have $\Phi - s^T(k)s(k) \prec \beta s^T(k)s(k)$.

Thus, by tuning the parameter q and ϵ, we have, $\Delta V_s(k) \prec \beta s^T(k)s(k)$ which guarantees the convergence of $\Delta V_s(k)$ and implies that any trajectory of the system (3.3,3.4) will be driven onto the sliding surface and maintain on it.
This completes the $\mathbf{proof}$.

The control signal $u(k)$ computed in (3.24) using compensated sliding surface (3.10), will also experience controller to actuator fractional delay (τ'_{ca}) which results in the delayed control signal $u(k-\tau'_{ca})$. So, in order to avoid the degradation of the plant response again the time delay is compensated from controller to actuator. The compensated control signal at the actuator end can be represented as:

$$u_a(k) = u(k) - \alpha' u(k-1), \tag{3.33}$$

where,

$\alpha' = \frac{\tau'_{ca}}{1+\tau'_{ca}}$.

It can be noticed from above Eqn. (3.33) that the compensated control signal $u_a(k)$ depends on difference of the present control signal that is available from network as well as past control signal which is multiplied over the parameter α' approximated through Thiran Approximation. Thus the effect of controller to actuator fractional delay is compensated at actuator side which is further applied to the plant.

3.6 Results and Discussions

This section briefly discusses about the simulation results as well as experimental results of the proposed control algorithm in the presence of deterministic network delays and matched uncertainty. The efficiency and robustness of the proposed control algorithm is tested considering DC Servo Motor as real time plant. On real time implementation the effects of control algorithm are examined in the presence of network non-idealities (fractional time delay and matched uncertainty).

3.6.1 System Description

In this section, Quanser Qnet 2.0 brushed DC motor setup is explained in detailed on which the simulation as well as experimental results are carried out using proposed control law. The performance of the Brushless DC Motor is tested under different networked delays as well external disturbances to prove the robustness of the proposed algorithm. The results obtained with proposed algorithm is compared with the conventional SMC algorithm.

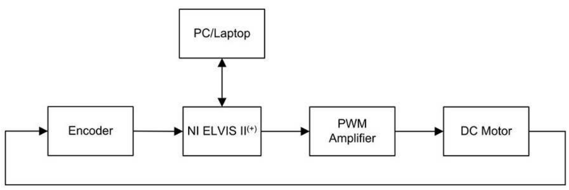


Figure 3.4: Block Diagram of Qnet DC Servo Motor Components

Figure (3.4) shows the block diagram of Quanser made Qnet 2.0 brushed DC motor setup used for the simulation as well as experimental purpose. The QNET DC Motor provides an integrated amplifier and a communication interface with the NI ELVIS II (+) for the amplifier command and encoder port. The NI ELVIS II (+) is interfaced to the PC via USB link to the QNET DC Motor setup as shown in Figure (3.5). The

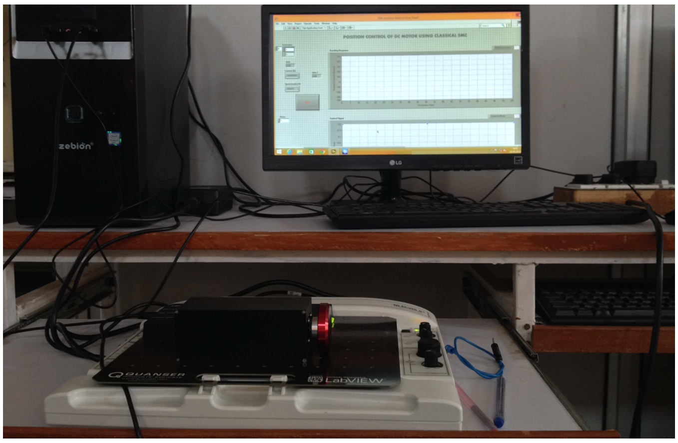


Figure 3.5: Experimental Setup of Quanser DC Servo Motor

NI ELVIS II (+) block reads the angular encoder as an input and commands the power amplifier which acts as driver for the motor. The various network delays are generated through software blocks.

The detailed mathematical model along with the system parameters of the DC motor are given as [Astrom *et al.* (2015)]:

$$\frac{\theta(s)}{V_m(s)} = \frac{K_m}{J_m R_m s^2 + K_m^2 s}, \tag{3.34}$$

where,

$\theta(s)$=output from the system (position),

V_m=input to the system,

J_m=Rotor inertia=$4 * 10^{-6} kgm^2$,

R_m=Terminal Resistance=8.4 ohms,

K_m=Motor back emf constant=0.042 V/(rad/s).

Substituting these parameters, the state space model of the above system (3.34) is given as,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dd(t), \tag{3.35}$$

$$y(t) = Cx(t), \tag{3.36}$$

where,

$A = \begin{bmatrix} -201 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$

$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

Discretizing the system at sampling interval $h = 30msec$, we get,

$$x(k+1) = Fx(k) + Gu(k - \tau') + d(k), \tag{3.37}$$

$$y(k) = Cx(k), \tag{3.38}$$

where,

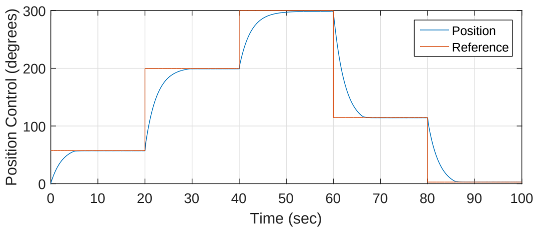
$F = \begin{bmatrix} 0.001836 & 0 \\ 0.004753 & 1 \end{bmatrix}, G = \begin{bmatrix} 0.004753 \\ -0.0001242 \end{bmatrix},$

$C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$

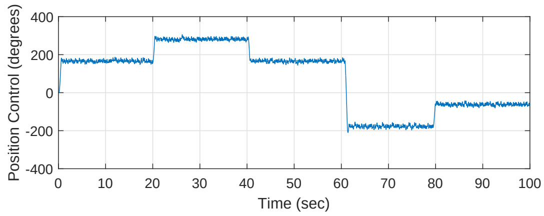
3.6.2 Simulation and Experimental Results of Brushless DC Motor

In this section, the simulation and experimental results of Position Control DC motor are thoroughly discussed in presence of deterministic network delays. Figures (3.6) to (3.10) shows the simulation and experimental responses of system for trajectory tracking, compensated sliding variable and control efforts under different network delays. To show the robustness properties slow time varying disturbance signal is applied through the input channel. The total networked induced delay with a range of $12.8msec$ to $28msec$ is generated through network block for which the effect of delay is compensated satisfying condition (3.5). The position of DC motor is considered as the reference input. The sliding gain is computed using discrete LQR method which comes out $C_s = [2.5156 \quad 31.6288]$ with $Q = diag(1000, 1000)$ and $R = 1$. While the quasi-sliding band (3.21) comes out to be $|s(k)| \leq -5$ to 5 with tunning parameters $q = 30$ and $\epsilon = 2000$.

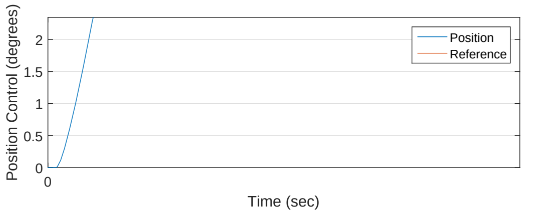
Figures (3.6(a)) to (3.7(d)) shows the simulation and experimental results of position control DC motor plant for total network delay of $\tau = 12.8msec$ with $\tau_{sc} = 6.4msec$ and $\tau_{ca} = 6.4msec$. The fractional part of total network delay is obtained as $\tau' = 0.426$, $\tau'_{sc} = 0.213$ and $\tau'_{ca} = 0.213$ for $h = 30msec$. The trajectory response of the system in case of simulation and experimental are shown in Figures (3.6(a)) and (3.6(b)) respectively. In both cases, the output tracks the reference trajectory in the presence of specified network delay. In order to show the exact effect of time delay compensation



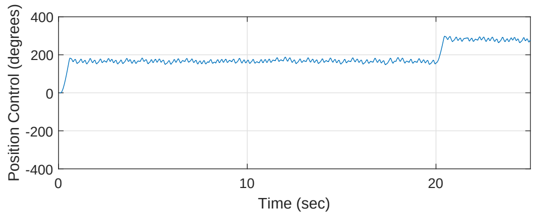
(a) Simulated response of Position control for τ=12.8msec



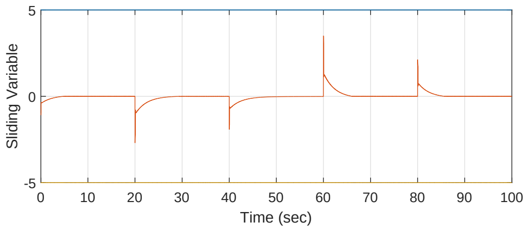
(b) Experimental response of Position control for τ=12.8msec



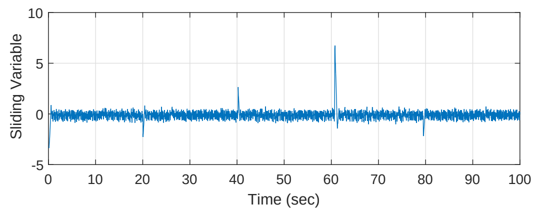
(c) Magnified simulated response of Position control for τ=12.8msec



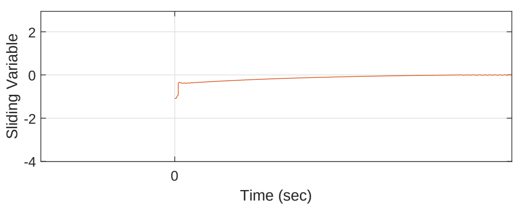
(d) Magnified experimental response of Position control for τ=12.8msec



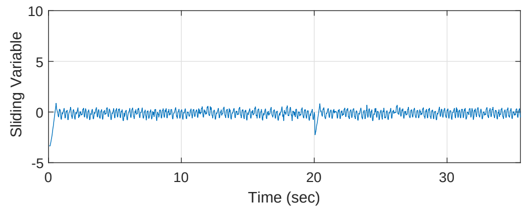
(e) Simulated compensated sliding variable for τ=12.8msec



(f) Experimental compensated sliding variable for τ=12.8msec

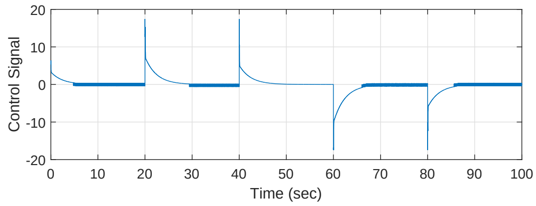


(g) Magnified simulated response of $s(k)$ for τ=12.8msec

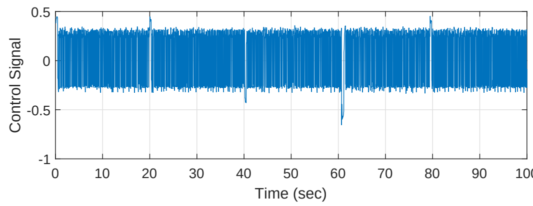


(h) Magnified experimental response of $s(k)$ for τ=12.8msec

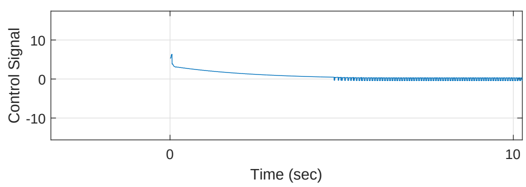
Figure 3.6: Simulation as well as Experimental results for tracking and compensated sliding surface for τ=12.8msec.



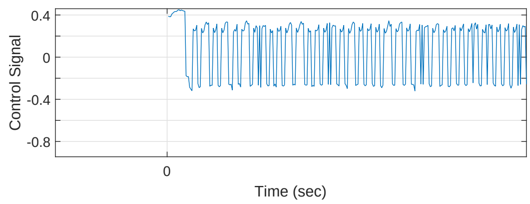
(a) Simulated control efforts for τ=12.8msec



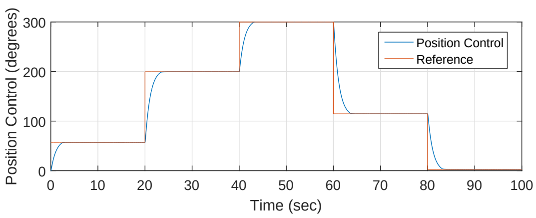
(b) Experimetal control efforts for τ=12.8msec



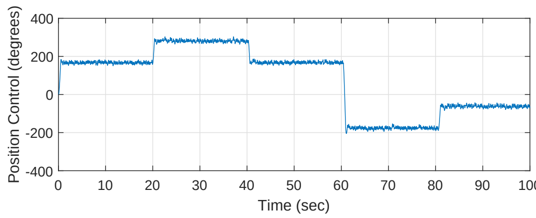
(c) Magnified simulated control efforts for τ=12.8msec



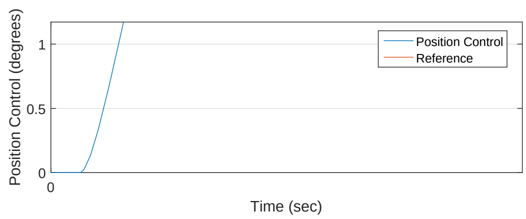
(d) Magnified experimental control efforts for τ=12.8msec



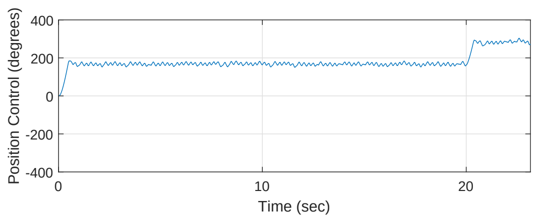
(e) Simulated response of Position control for τ=24msec



(f) Experimental response of Position control for τ=24msec

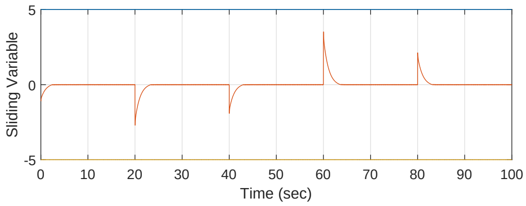


(g) Magnified simulated response of Position control for τ=24msec

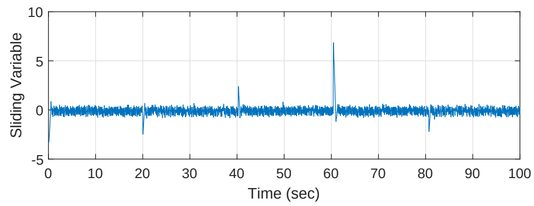


(h) Magnified experimental response of Position control for τ=24msec

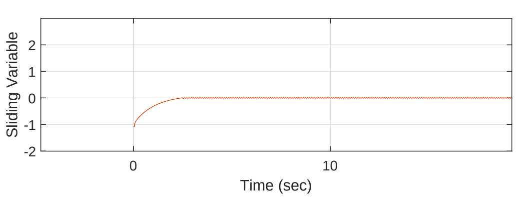
Figure 3.7: Simulation as well as Experimental results of control signal and tracking response for τ=12.8msec and τ=24msec.



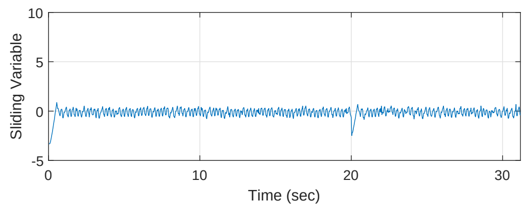
(a) Simulated compensated sliding variable for τ=24msec



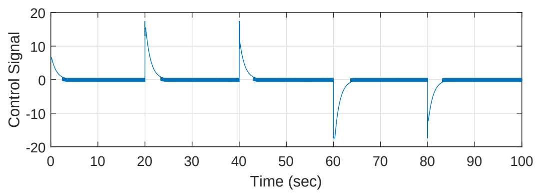
(b) Experimental compensated sliding variable for τ=24msec



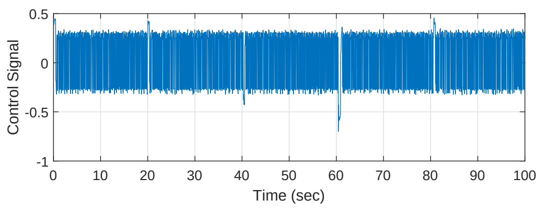
(c) Magnified simulated response of $s(k)$ for τ=24msec



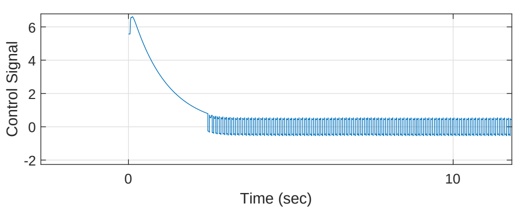
(d) Magnified experimental response of $s(k)$ for τ=24msec



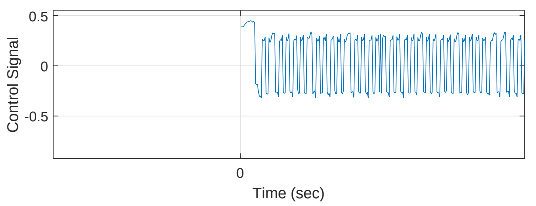
(e) Simulated control efforts for τ=24msec



(f) Experimental control efforts for τ=24msec

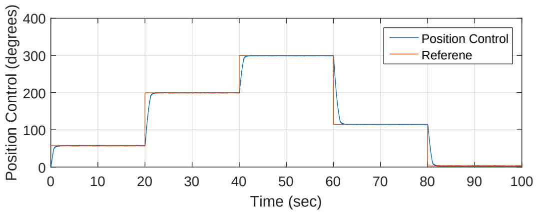


(g) Magnified simulated control efforts for τ=24msec

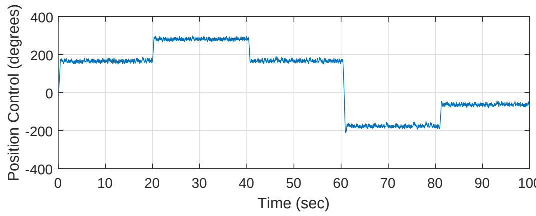


(h) Magnified experimental control efforts for τ=24msec

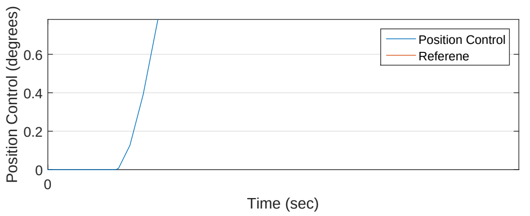
Figure 3.8: Simulation as well as Experimental results of compensated sliding surface and control signal for τ=24msec.



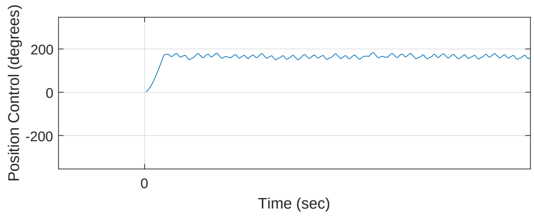
(a) Simulated response of Position control for τ=28msec



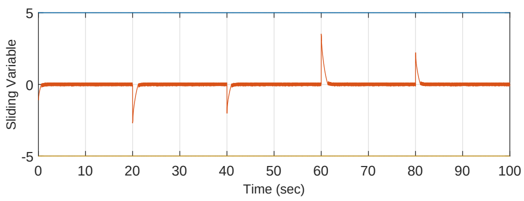
(b) Experimental response of Position control for τ=28msec



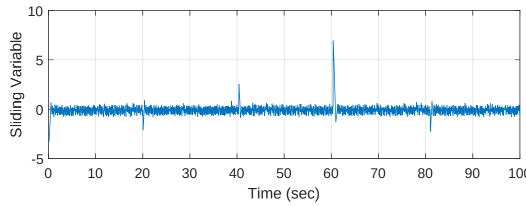
(c) Magnified simulated response of Position control for τ=28msec



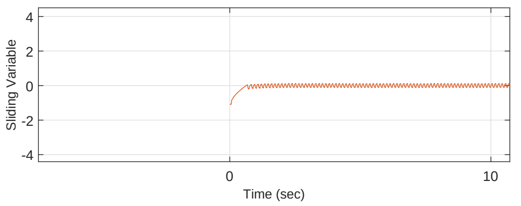
(d) Magnified experimental response of Position control for τ=28msec



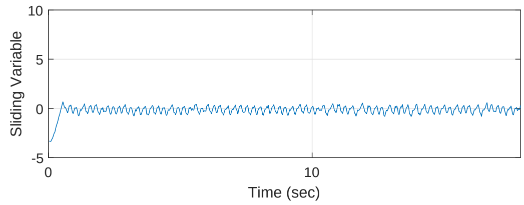
(e) Simulated compensated sliding variable for τ=28msec



(f) Experimental compensated sliding variable for τ=28msec

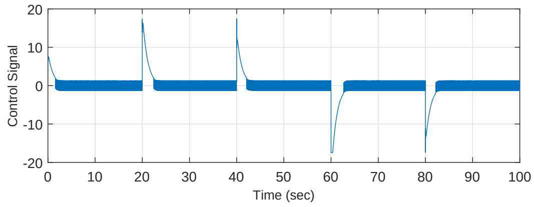


(g) Magnified simulated response of $s(k)$ for τ=28msec

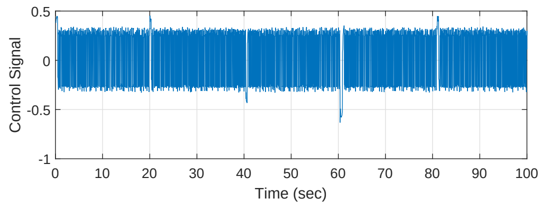


(h) Magnified experimental response of $s(k)$ for τ=28msec

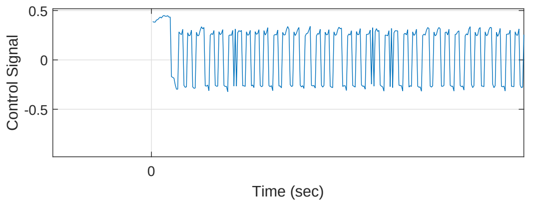
Figure 3.9: Simulation as well as Experimental results of tracking and compensated sliding surface for τ=28msec.



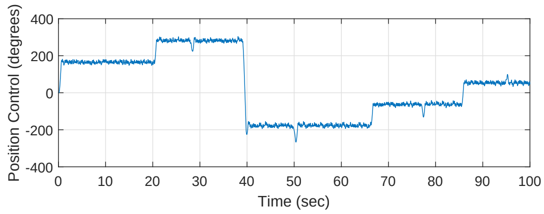
(a) Simulated control efforts for τ=28msec



(b) Experimental control efforts for τ=28msec



(c) Magnified control efforts for τ=28msec



(d) Experimental tracking response of Position control under external disturbance

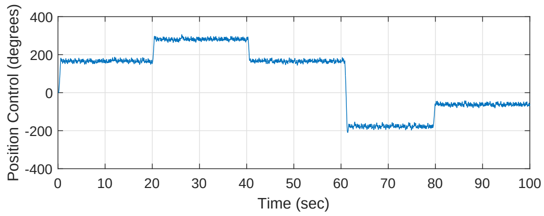
Figure 3.10: Simulation as well as Experimental results of control efforts for τ=28msec along with tracking response under external disturbances.

at the output, results are magnified as shown in Figures (3.6(c)) and (3.6(d)). It can be noticed that the effect of fractional time delay from sensor to controller is compensated as the output tracks the trajectory at $6.4msec$. The same effect of time delay compensation from sensor to controller can be observed in sliding surface Figures (3.6(e)) and (3.6(f)) as well as control signal Figures (3.7(a)) and (3.7(b)). Observing the magnified results of Figures (3.6(g)), (3.6(h)), (3.7(c)) and (3.7(d)) it can be noticed that the sliding surface and control signal both are computed at first sampling instant even in the presence of sensor to controller delay. Thus, the effects of fractional delay from sensor to controller at sliding surface and control signal are compensated and remains within the specified sliding band (3.21). The proposed algorithm was further extended for higher values of τ. Figures (3.7(e)) to (3.8(h)) shows the simulation and experimental results of position control DC motor for total networked delay of $\tau = 24msec$ with $\tau_{sc} = 12msec$ and $\tau_{ca} = 12msec$. The fractional part of total network delay is computed as $\tau' = 0.8$, $\tau'_{sc} = 0.4$ and $\tau'_{ca} = 0.4$ for $h = 30msec$. The simulated and experimental trajectory response of the system are shown in Figures (3.7(e)) and (3.7(f)) respectively. Observing the results it can be noticed the output tracks the reference signal in the specified networked delay. In order to show the effect of delay compensation the output results are magnified shown in Figures (3.7(g)) and (3.7(h)) respectively. It can be noticed that the effect of fractional delay from sensor to controller is nullified as the output tracks the reference trajectory at $t = 12msec$. The similar effect of time delay compensation can be observed in sliding surface as well as control efforts signal shown in Figures (3.8(a)) to (3.8(h)). Observing the simulated and experimental magnified results of sliding surface [(3.8(c)) and (3.8(d))] as well as control signal [(3.8(g)) and (3.8(h))], it can be noticed that, in both the cases the sliding surface and control signal are computed at first sampling instant. Thus the fractional delay from sensor to controller is compensated and remains within the specified sliding band (3.21). Figures (3.9(a)) to (3.10(c)) shows the simulation and experimental results of position control DC motor for total networked delay of $\tau = 28msec$ with $\tau_{sc} = 14msec$ and $\tau_{ca} = 14msec$. The fractional part of total networked delay for $h = 30msec$ is obtained as $\tau' = 0.933$, $\tau'_{sc} = 0.466$ and $\tau'_{ca} = 0.466$ respectively. The simulation and experimental results with magnified response of reference trajectory are shown in Figures (3.9(a)) to (3.9(d)) respectively. Observing the results it can be concluded that the output tracks the reference trajectory at $t = 14msec$ for the specified networked delay. Thus the effect of fractional delay from sensor to controller is nullified at the output as shown in Figures (3.9(c)) and (3.9(d)). The similar effect of time delay compensation will be observed in sliding surface and control signal results shown in Figures (3.9(e)) to (3.10(c)). Observing the results it can be noticed that in simulation as well as experimental case the sliding surface and the control signal are computed from first sampling instant. Thus the effect of fractional delay from sensor to controller is compensated at sliding surface as well as at control signal. Apart from delay compensation, the position of motor was also controlled by applying the external disturbances through rotating the wheel in forward and reverse direction. The situation of motor under external disturbances is shown in Figure (3.10(d)).

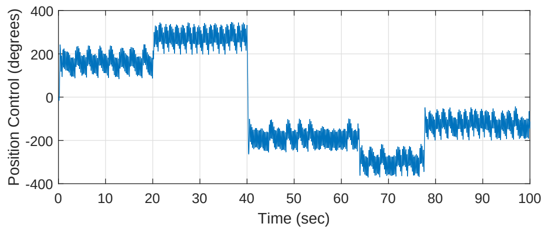
Thus, from all the results it can be concluded that the proposed algorithm works efficiently with network delay range of $12.8msec \leq \tau \leq 28msec$ in experimental as well as in simulated enviornment. The proposed controller compensates the network time delay for $q = 30$ and $\epsilon = 2000$ satisfying (3.5) and shows the stable response satisfying condition (3.25) in the presence of matched uncertainty.

Comparison of proposed algorithm with conventional sliding mode control

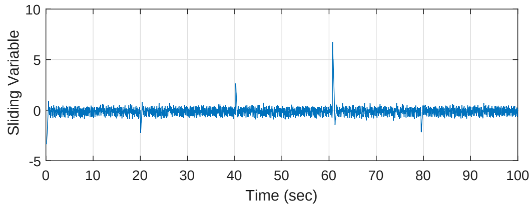
In this section, the experimental results of proposed algorithm are compared with conventional sliding mode control. The sliding surface and control algorithm for conven-



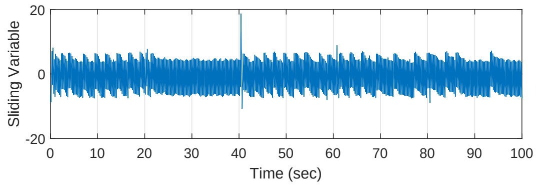
(a) Position control of DC motor using proposed algorithm for τ=12.8msec



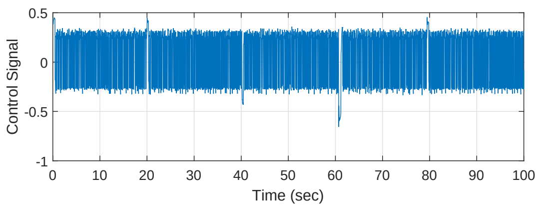
(b) Position control of DC motor using conventional SMC algorithm for τ=12.8msec



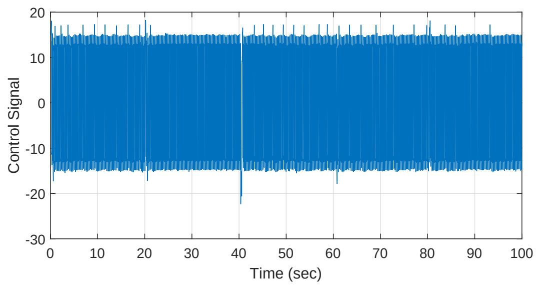
(c) Compensated sliding variable of proposed algorithm for τ=12.8msec



(d) Compensated sliding variable of conventional SMC algorithm for τ=12.8msec



(e) Control efforts response of proposed algorithm for τ=12.8msec



(f) Control efforts response of conventional SMC algorithm for τ=12.8msec

Figure 3.11: Comparison of proposed algorithm and conventional sliding mode control for τ=12.8msec

tional sliding mode control is defined as:

$$s_c(k) = C_s x(k - \tau'_{sc}),$$

$$u(k) = -(C_s G)^{-1}[C_s x(k - \tau'_{sc}) - (1 - qh)(s_c(k)) + \epsilon h sgn(s_c(k))] - d(k - \tau'_{sc}).$$

The results of tracking response, control signal and sliding variable for total networked delay of $\tau = 12.8msec$ are shown in Figures (3.11(a)) to (3.11(f)). From the comparative results it can be noticed that the conventional sliding mode control becomes unstable for a small delay of $\tau_{sc} = 6.4msec$. Thus, the Thiran's approximation proved to be efficient method in discrete-time sliding mode control. Simulation and experimental results for different networked delays range are summarized in Table-3.1. The

Network Delays (τ)	Simulations or Experimental	
	$Chattering$	$Design\ performance$
$12.8msec$	within QSMB	satisfactory
$24msec$	within QSMB	satisfactory
$28msec$	within QSMB	satisfactory

Table 3.1: Simulations and Experimental Results Under different Networked Delays

Algorithm	Comparison Results			
	τ	T_s	$Chattering$	$Response$
$Coventional\ SMC$	12.8msec	Undefined	high	unstable
$Proposed\ method$	12.8msec	1sec	within QSMB	stable

Table 3.2: Comparison of Proposed Algorithm with Conventional SMC

comparison of discrete-time sliding mode control with time delay approximation and conventional sliding mode control are shown in Table- 3.2.

3.7 Conclusion

In this chapter, we explored Thiran's approximation technique for fractional delay compensation in discrete-time domain. The effect of fractional delay generated due to the communication medium is compensated in sliding surface. The sliding surface is designed in such a manner that the system states slides along the predetermined surface according to network delay. Using this approach, switching type discrete-time sliding mode controller is designed which computes the control actions in the presence of network delay and matched uncertainty. The stability of the closed loop NCS is assured by using Lyapunov approach. The effectiveness of the derived algorithm is tested on brushless DC motor setup with deterministic networked delay and matched uncertainty. The experimental results are compared with conventional SMC algorithm. The comparative results proved that the fractional delay approximated using Thiran Approximation is most efficient technique as it compensates the fractional network delays in discrete-time domain in the presence of matched as well as unmatched uncertainties.

CHAPTER 4

DESIGN OF DISCRETE-TIME SLIDING MODE CONTROLLER (NON-SWITCHING TYPE) FOR FRACTIONAL DELAY

4.1 Introduction

In this chapter, a unique approach is presented for designing non-switching discrete-time sliding mode controller using Thiran's delay approximation. The effect of sensor to controller delay is compensated using Thiran's delay approximation technique in sliding surface. Further, Lyapunov approach is used to determine the stability of closed loop NCSs with proposed controller. The feasibility and effectiveness of the control methodology are outlined through simulation and experimental results showing the siginificant response in the presence of networked delay. The efficacy of the proposed control algorithm is further validated in the presence of real time networks such as CAN and Switched Ethernet using True Time Simulator.

4.2 Problem Formulation

$\mathbf{Remark-4}$: In this chapter nature of the system, total fractional network delays (τ'), sensor to controller fractional delay (τ'_{sc}) and controller to actuator fractional delay (τ'_{ca}) would remain same as mentioned in section (3.3).

$\mathbf{Problem\ Statement}$: The main objective, is to design non-switching based discrete-time sliding mode control law for system (3.3,3.4) in the presence of fractional delay τ'_{sc} and τ'_{ca} satisfying (3.5) and matched uncertainty satisfying (3.7).

The next section describes the design of non-switching type sliding mode control law using the proposed sliding surface (3.18).

4.3 Discrete-Time Sliding Mode Control (Non-Switching type) Using Thiran's Delay Approximation

In this section, non-switching type control law alongwith its stability is proposed based on reaching law in [Bartoszewicz and Lesniewski (2014)] using compensated sliding surface (3.18). The reaching law proposed by Bartoszewicz causes less chattering compare to Gao's law and offers faster covergence with limited magnitude of the control signal.

$\mathbf{Theorem-2}$: The non-switching discrete-time sliding mode controller for system $(3.3, 3.4)$ in the presence of deterministic sensor to controller fractional delay satisfying (3.5) and matched uncertainty $d(k)$ is given as,

$$u(k) = -(C_sG)^{-1}[Hx(k) - Ix(k) - J(s(k)) + d_s(k) - d_1] - d(k). \tag{4.1}$$

where,

$H = (C_sF)$, $I = \alpha C_s$, $J = \{1 - q[s(k)]\}$.

$\mathbf{Proof}$: Let us consider the reaching law in [Bartoszewicz and Lesniewski (2014)] in the presence of sensor to controller fractional delay given as:

$$s[(k+1)h] = \{1 - q[s(k)]\} - d_s(k) + d_1, \tag{4.2}$$

where,
where $d_s(k)$ is disturbance at the controller end, $q[s(k)] = \frac{\psi}{\psi+|s(k)|}$ with ψ as user defined constant satisfying $\psi \geq d_2$, d_1 and d_2 are mean and deviation of $d(k)$.
$\mathbf{Remark-5}$: The disturbace $d(k)$ appearing in the reaching law is applied through the

network. So, the compensated disturbance using Thiran's approximation is given as:

$$d_s(k) = d(k) - \alpha d(k-1). \tag{4.3}$$

The reaching law in Eqn. (4.2) indicates that the system states always move towards the specified sliding band given as:

$$|s(k)| \leq \frac{\psi d_2}{\psi - d_2}. \tag{4.4}$$

Substituting Eqn. (3.18) into Eqn. (4.2) we may get:

$$C_s x(k+1) - \alpha C_s x(k) = \{1 - q[s(k)]\} - d_s(k) + d_1,$$

and substituting the value of $x(k+1)$ we get,

$$C_s[Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k) = \{1 - q[s(k)]\} - d_s(k) + d_1.$$

Further simplifying we may write,

$$C_s F x(k) + C_s G(u(k) + d(k)) - \alpha C_s x(k) = \{1 - q[s(k)]\} - d_s(k) + d_1. \tag{4.5}$$

Further, solving the above Eqn. (4.5), control law can be expressed as:

$$u(k) = -(C_s G)^{-1}[Hx(k) - Ix(k) - J(s(k)) + d_s(k) - d_1] - d(k). \tag{4.6}$$

where,

$H = (C_s F)$, $I = \alpha C_s$ and $J = \{1 - q[s(k)]\}$
This completes the $\mathbf{proof}$.

The stability condition is derived further using compensated sliding surface (3.18) and control law proposed in Eqn. (4.6) such that the system states remain within specified band (4.4) over a finite interval of time.

4.3.1 Stability Analysis

For given positive scalars τ'_{sc} and τ'_{ca} with total networked delay τ' satisfying (3.5), the trajectories of the closed loop system (3.3,3.4) with controller (4.6) and $d(k)$ satisfying (3.7) drive towards the sliding surface (3.18) such that the following condition (4.7) is feasible:

$$0 \preceq \kappa \prec s^T(k)s(k). \tag{4.7}$$

Proof: The compensated sliding surface is given by,

$$s(k) = C_s x(k) - \alpha C_s x(k-1). \tag{4.8}$$

Selecting the Lyapunov function as,

$$V_s(k) = s^T(k)s(k). \tag{4.9}$$

Writing forward difference of the above equation,

$$\Delta V_s(k) = s^T(k+1)s(k+1) - s^T(k)s(k). \tag{4.10}$$

Substituting the value of $s(k+1)$ using Eqn. (4.8) we get,

$$\begin{aligned}\Delta V_s(k) = [C_s x(k+1) - \alpha C_s x(k)]^T [2C_s x(k+1) \\ -\alpha C_s x(k)] - s^T(k)s(k).\end{aligned} \tag{4.11}$$

Substituting the value of $x(k+1)$,

$$\begin{aligned}\Delta V_s(k) = [C_s[Fx(k) + G(u(k) + d(k))] - \alpha C_s x(k)]^T \\ [C_s F x(k) + G(u(k) + d(k))] - \alpha C_s x(k)] - s^T(k)s(k).\end{aligned} \tag{4.12}$$

Substituting the value of $u(k)$ from Eqn. (4.6) and further solving it we have,

$$\begin{aligned}\Delta V_s(k) = [(1 - q[s(k)])s(k) - d_s(k) + d_1]^T * [(1 - q[s(k)]) \\ s(k) - d_s(k) + d_1] - s^T(k)s(k).\end{aligned} \tag{4.13}$$

Denoting,

$$\kappa = [(1 - q[s(k)])s(k) - d_s(k) + d_1]^T * [(1 - q[s(k)])s(k) - d_s(k) + d_1]$$

Then we have,

$$\Delta V_s(k) = \kappa - s^T(k)s(k). \tag{4.14}$$

The term κ can be tuned close to zero by appropriately selecting the parameter ψ. If κ is closed to zero, then $s^T(k)s(k)$ will be larger than κ. Thus, for any small parameter η, we have $\kappa - s^T(k)s(k) \prec \eta s^T(k)s(k)$.

Thus, by tuning the parameter ψ, we have, $\Delta V_s(k) \prec \eta s^T(k)s(k)$ which guarantees the convergence of $\Delta V_s(k)$ and implies that any trajectory of the system (3.3,3.4) will be driven onto the sliding surface and maintain on it.

This completes the $\mathbf{proof}$.

4.4 Results and Discussions

This section briefly discusses about the simulation results as well as experimental results of the proposed control algorithm in the presence of deterministic network delays and matched uncertainty. The efficiency and robustness of the proposed control algorithms are tested under three different situations: (i) illustrative example (ii) real time plant as DC servo motor and (iii) real time networks.

4.4.1 Simulation Results with Illustrative Example

In this segment an illustrative example from [Wu and Chen (2007)] is simulated in MATLAB environment.

Consider the continuous-time LTI system as,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dd(t), \tag{4.15}$$

$$y(t) = Cx(t), \tag{4.16}$$

where,

$A = \begin{bmatrix} -0.7 & 2 \\ 0 & -1.5 \end{bmatrix}$, $B = \begin{bmatrix} -0.03 \\ -1 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Discretizing the above system parameters at sampling interval of $h = 30msec$,

$$x(k+1) = Fx(k) + Gu(k-\tau') + d(k), \tag{4.17}$$

$$y(k) = Cx(k), \tag{4.18}$$

where,

$F = \begin{bmatrix} 0.9792 & 0.05805 \\ 0 & 0.956 \end{bmatrix}$, $G = \begin{bmatrix} -0.001771 \\ -0.02934 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

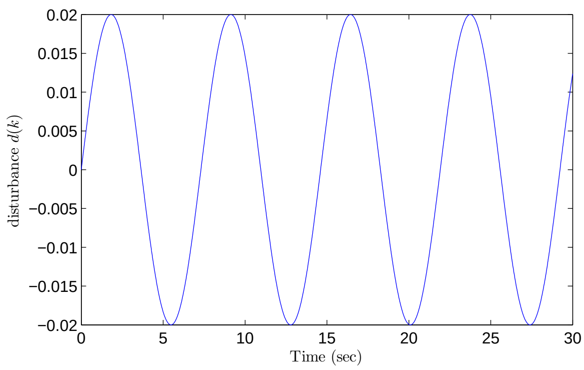


Figure 4.1: Slow time varying disturbance $d(k)$

Figures (4.2) to (4.13) shows the nature of the system under networked enviornment. In order to check the robustness of the derived control law a slow time varying distur-

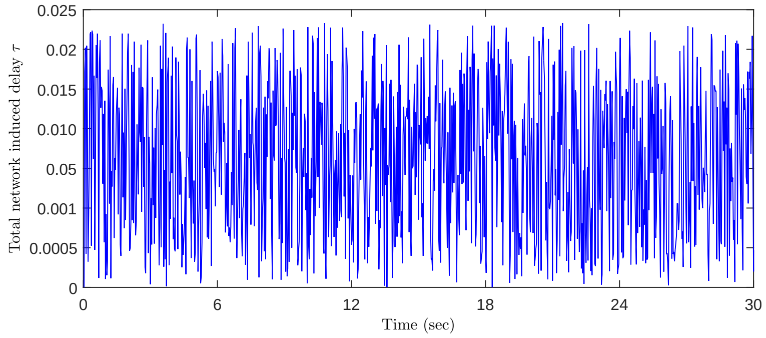


Figure 4.2: Total network delay τ

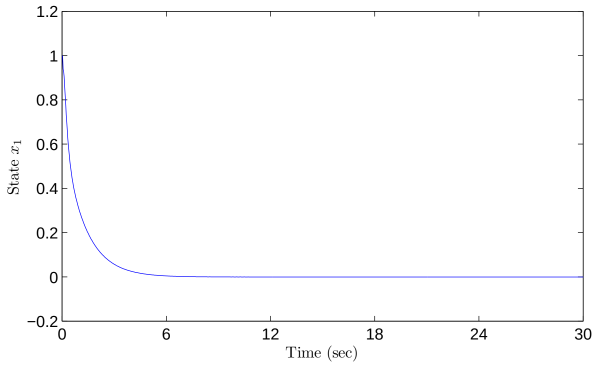


Figure 4.3: State variable x_1 with initial condition x_1=1

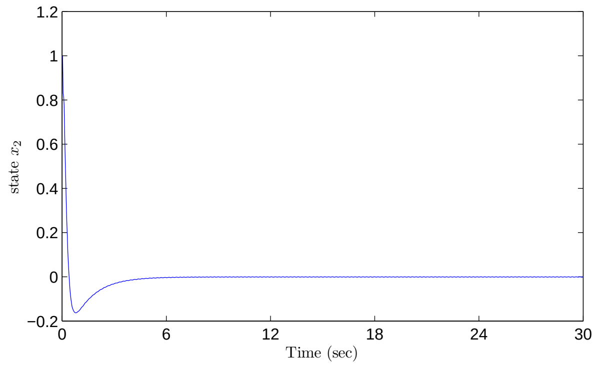


Figure 4.4: State variable x_2 with initial condition x_2=1

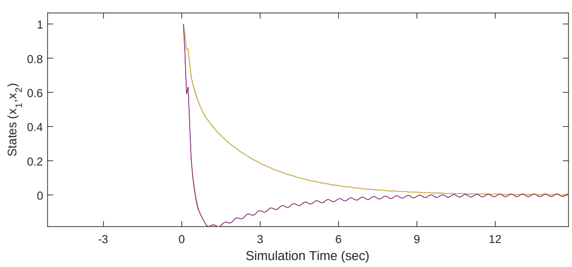


Figure 4.5: Magnified result of state variables x_1, x_2

bance is applied at the input of the system as shown in Figure (4.1). Figure (4.2) shows the deterministic network induced fractional delay with range of $3msec \leq \tau \leq 20msec$

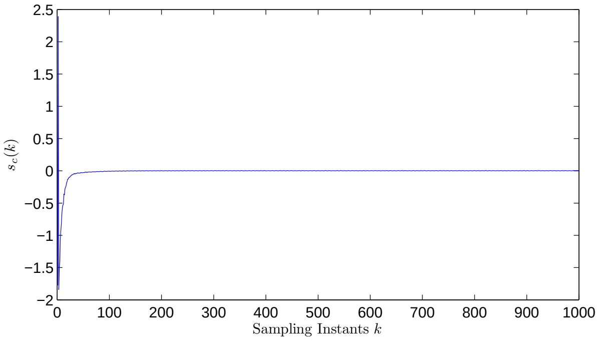


Figure 4.6: Compensated sliding surface $s(k)$

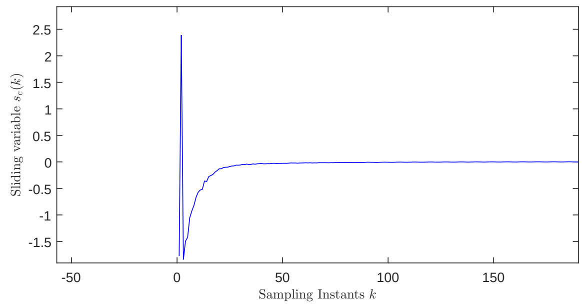


Figure 4.7: Magnified result of compensated sliding surface $s(k)$

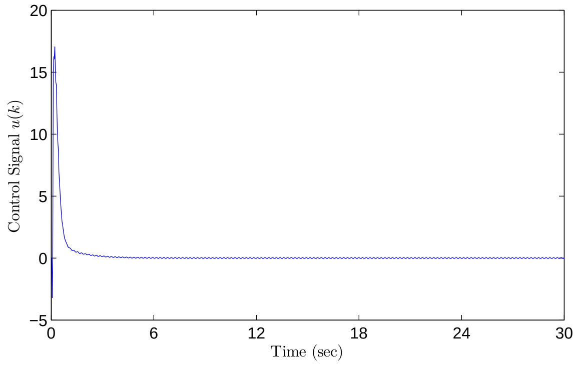


Figure 4.8: Control signal $u(k)$

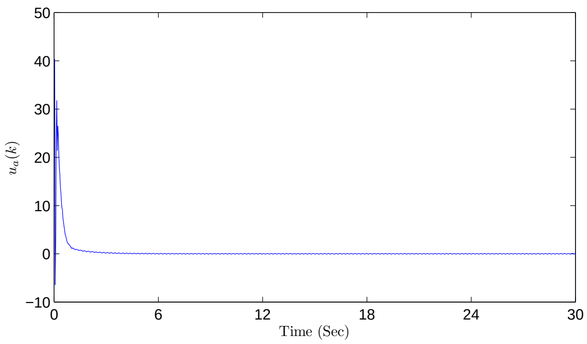


Figure 4.9: Compensated control signal $u_a(k)$

under which the system shows the stable response satisfying (3.5). In this work, network delay is considered as the time required for the data packets to travel from sensor

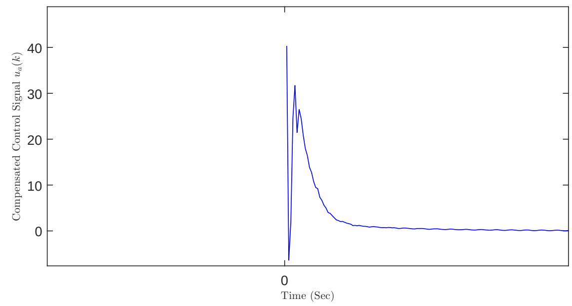


Figure 4.10: Magnified compensated control signal $u_a(k)$

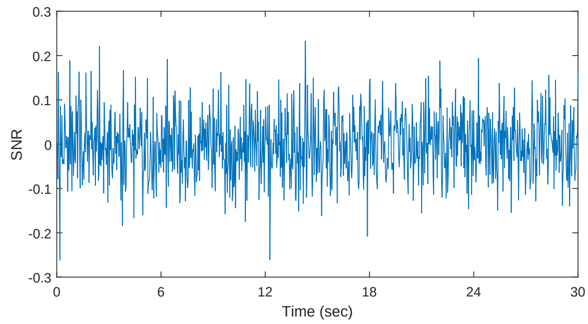


Figure 4.11: Response of SNR

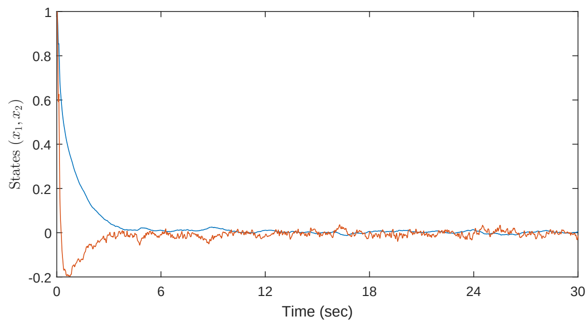


Figure 4.12: Nature of state variables for different SNR

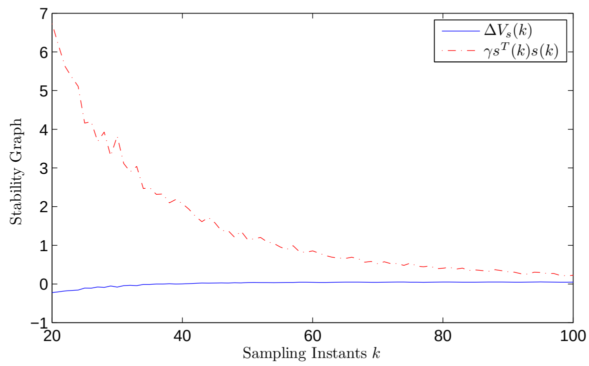


Figure 4.13: Result of Stability

to controller and controller to actuator. The time required for data packets to travel from sensor to controller is $1.5msec \leq \tau_{sc} \leq 10msec$ and for controller to actuator is

$1.5msec \leq \tau_{ca} \leq 10msec$ respectively. The sliding gain C_s is calculated using discrete LQR method with $Q = diag(1000, 1000)$ and $R = 1$. The computed values of sliding gain are $C_s = [-1.77 \quad -2.766]$. The quasi-sliding mode band computed to be $|s(k)| \leq +0.2$ to -0.2 with proper selection of user defined constant $\psi = 100$.

Figures (4.3) and (4.4) shows the plant state variables with initial condition $x(k) = [1 \quad 1]$. Both the states converges to zero from given initial condition in the presence of network fractional delay. Figure (4.5) shows the magnified result of the plant state variables. It can be noticed that the effect of network fractional delay at the ouptut of the system is compensated as it is computed from first sampling instant. Figure (4.6) shows the compensated sliding surface calculated using Thiran approximation rule. It can be observed that the compensated sliding variable is computed from first sampling instant even in the presence of sensor to controller fractional delay. The magnified response of the same is shown in Figure (4.7). Figure (4.8) shows the control signal $u(k)$ which is computed using proposed compensated sliding surface $s(k)$. This control signal is further applied to the plant through the network. The same approach of time delay compensation is used to compute the compensated control signal $u_a(k)$. The result of the same is shown in Figure (4.9). From the magnified result in Figure (4.10), it can be justified that the effect of controller to actuator fractional delay is also compensated as the control signal is computed from first sampling interval.

The algorithm is also examined for different SNR as shown in Figure (4.11). It can be observed from Figure (4.12) that the system states coverge to zero for different SNR. Figure (4.13) shows the results of stability. It can be observed from Figure (4.13) that for given $\psi = 100$ and $d_2 = 0.2$ guarantees the covergence of $\Delta V_s(k)$ and implies that the trajectories of system (3.3,3.4) will be driven on the compensated sliding surface and maintain on it under the specified network fractional delay and matched uncertainty. Thus, from above results it is justified that Thiran approximation provides better compensation in discrete-time domain in the presence of deterministic network delays and matched uncertainty.

4.4.2 Simulation and Experimental Results of Brushless DC Motor

The state space model of the system (3.34) is given as,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dd(t), \tag{4.19}$$

$$y(t) = Cx(t), \tag{4.20}$$

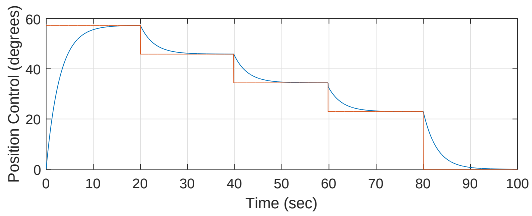
where,
$A = \begin{bmatrix} -201 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
Discretizing the system at sampling interval $h = 30msec$, we get,

$$x(k+1) = Fx(k) + Gu(k-\tau') + d(k), \tag{4.21}$$

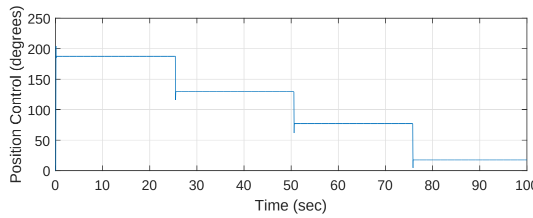
$$y(k) = Cx(k), \tag{4.22}$$

where,
$F = \begin{bmatrix} 0.001836 & 0 \\ 0.004753 & 1 \end{bmatrix}$, $G = \begin{bmatrix} 0.004753 \\ -0.0001242 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

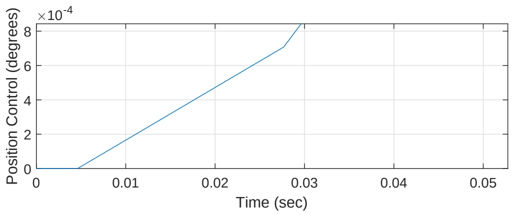
This section briefs about the simulation and experimental results of Position control Brushless DC motor in the presence of various deterministic delays using non-switching control law. The effect of time delay compensation are deeply analyzed through tracking response, compensated sliding variable and control signal for different network delays as shown in Figures (4.14) to (4.18). The robustness of the proposed algorithm is determined by applying time varying disturbance signal at the input side of the channel. The total networked induced delay with a range of $10msec$ to $28msec$ was generated for which the effect of time delay is compensated satisfying condition (3.5). The sliding gain is computed through discrete LQR method which comes out to be $C_s = [24.5156 \quad 31.6288]$ with $Q = diag(1000, 1000)$ and $R = 1$. The quasi-sliding mode band computed to be $|s(k)| \leq +0.2$ to -0.2 with proper selection of user defined constant $\psi = 1500$. Figures (4.14(a)) to (4.15(d)) shows the simulated and experimental results of position control brushless DC motor for total networked induced



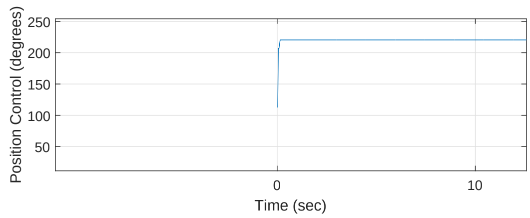
(a) Simulated result of Position control for τ=10msec



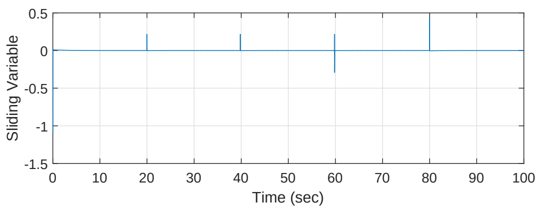
(b) Experimental result of Position control for τ=10msec



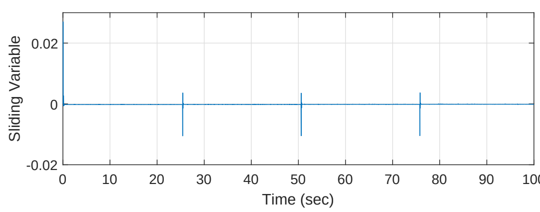
(c) Magnified simulated result of Position control for τ=10msec



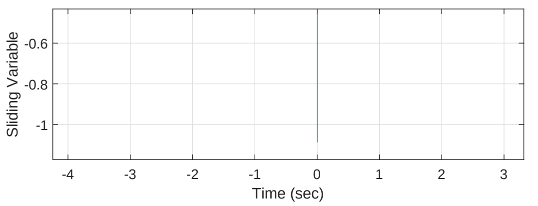
(d) Magnified experimental result of Position control for τ=10msec



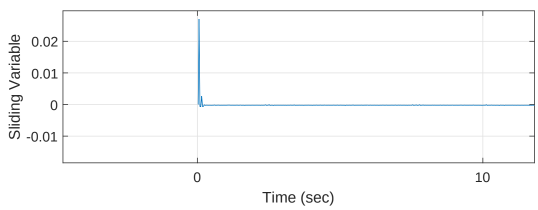
(e) Simulated result of compensated sliding variable for τ=10msec



(f) Experimental result of compensated sliding variable for τ=10msec

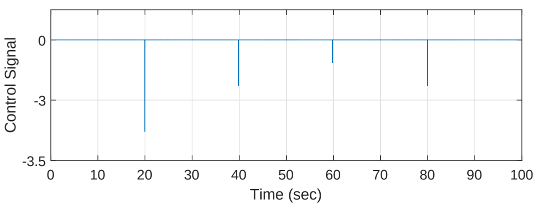


(g) Magnified simulated result of $s(k)$ for τ=10msec

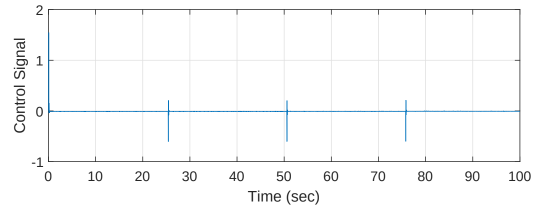


(h) Magnified experimental result of $s(k)$ for τ=10msec

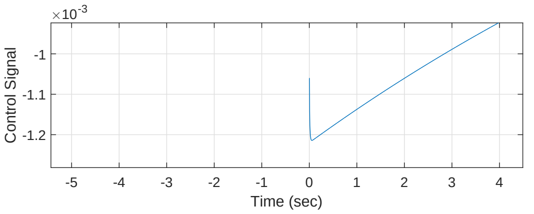
Figure 4.14: Simulation as well as Experimental results of tracking and compensated sliding surface for τ=10msec



(a) Simulated control signal for τ=10msec

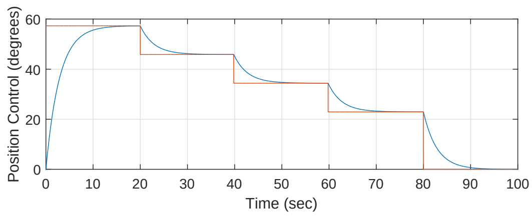


(b) Experimetal control signal for τ=10msec

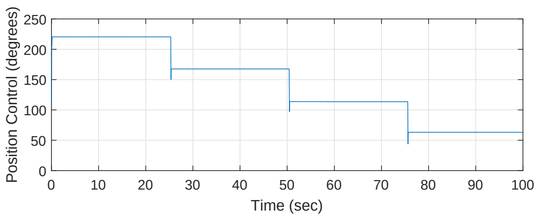


(c) Magnified simulated control signal for τ=10msec

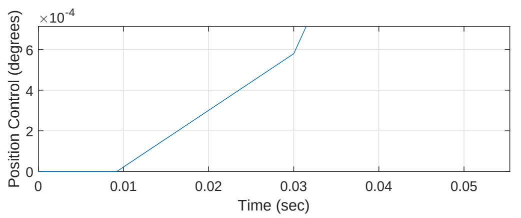
(d) Magnified experimental control signal for τ=10msec



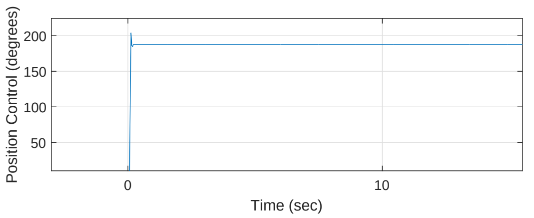
(e) Simulated result of Position control for τ=18msec



(f) Experimental result of Position control for τ=18msec

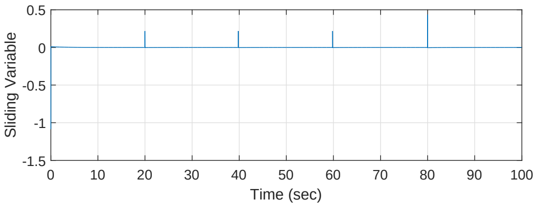


(g) Magnified simulated result of Position control for τ=18msec

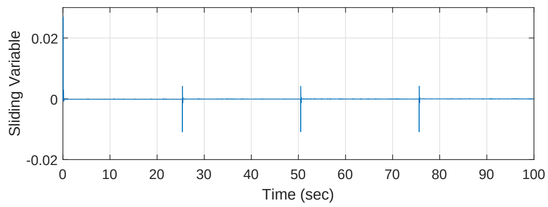


(h) Magnified experimental result of Position control for τ=18msec

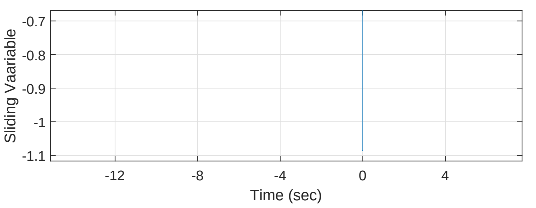
Figure 4.15: Simulation as well as Experimental results of control signal and tracking response for τ=18msec and τ=24msec



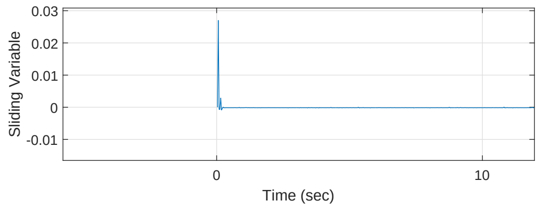
(a) Simulated result of compensated sliding variable for τ=18msec



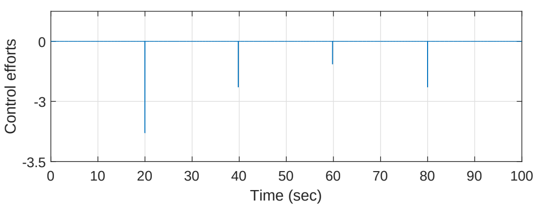
(b) Experimental result of compensated sliding variable for τ=18msec



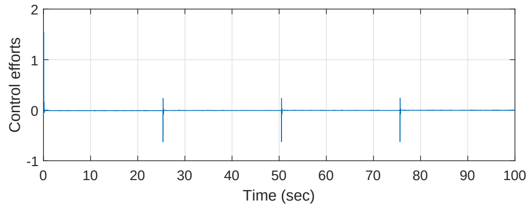
(c) Magnified simulated result of $s(k)$ for τ=18msec



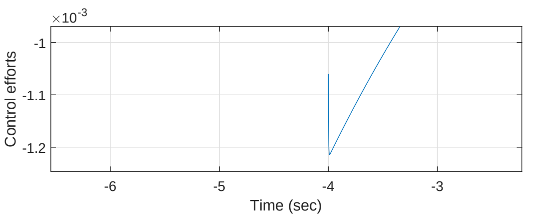
(d) Magnified experimental result of $s(k)$ for τ=18msec



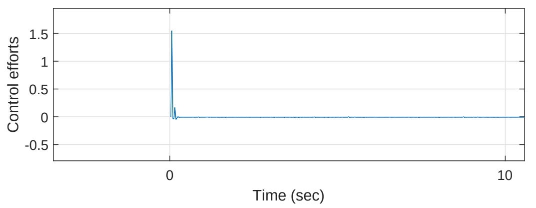
(e) Simulated control signal for τ=18msec



(f) Experimental control signal for τ=18msec

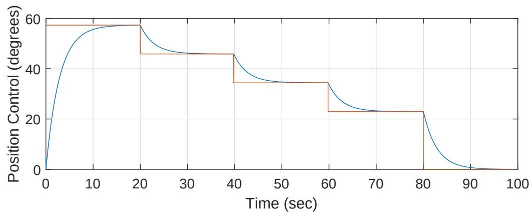


(g) Magnified simulated control signal for τ=18msec

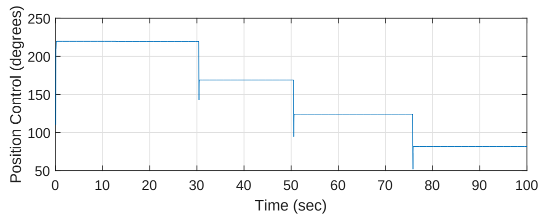


(h) Magnified experimental control signal for τ=18msec

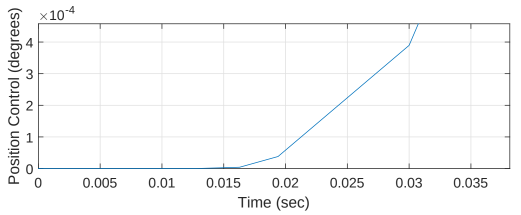
Figure 4.16: Simulation as well as Experimental results of compensated sliding surface and control signal for τ=18msec



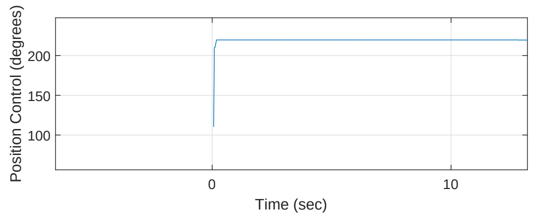
(a) Simulated result of Position control for τ=28msec



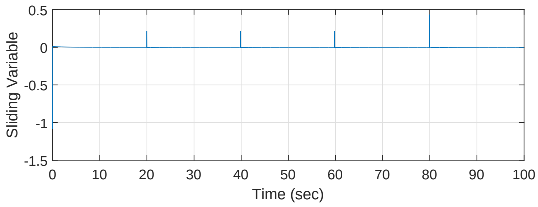
(b) Experimental result of Position control for τ=28msec



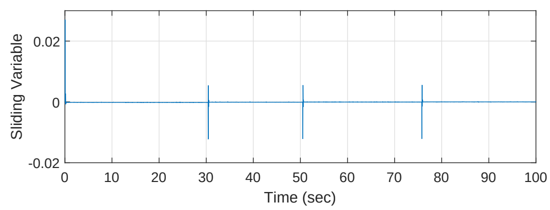
(c) Magnified simulated result of Position control for τ=28msec



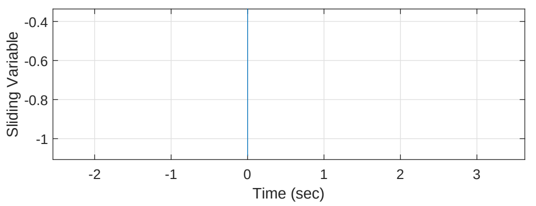
(d) Magnified experimental result of Position control for τ=28msec



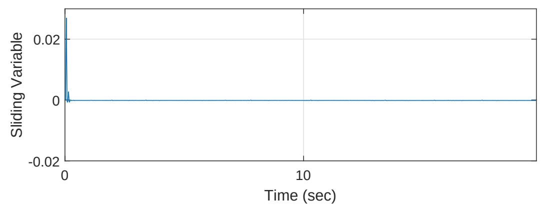
(e) Simulated result of compensated sliding variable for τ=28msec



(f) Experimental result of compensated sliding variable for τ=28msec

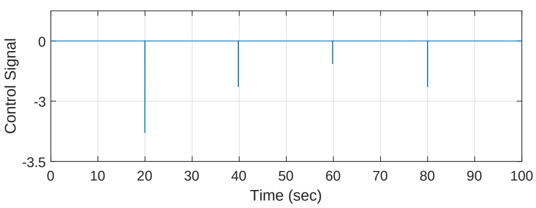


(g) Magnified simulated result of $s(k)$ for τ=28msec

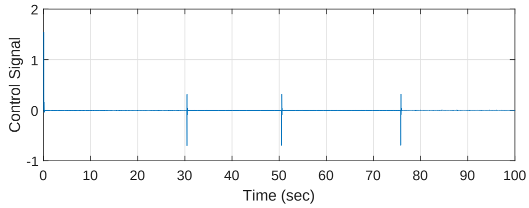


(h) Magnified experimental result of $s(k)$ for τ=28msec

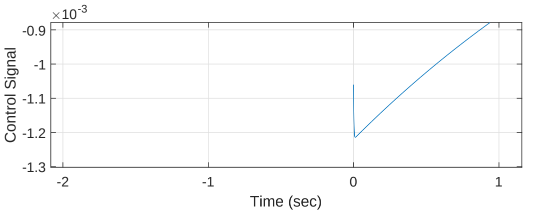
Figure 4.17: Simulation as well as Experimental results of tracking and compensated sliding surface for τ=28msec



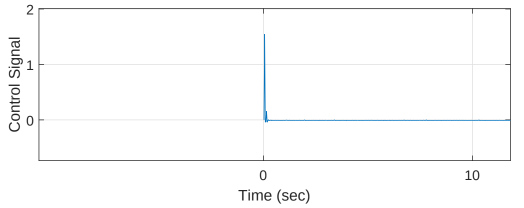
(a) Simulated control signal for τ=28msec



(b) Experimental control signal for τ=28msec



(c) Simulated Magnified control signal for τ=28msec

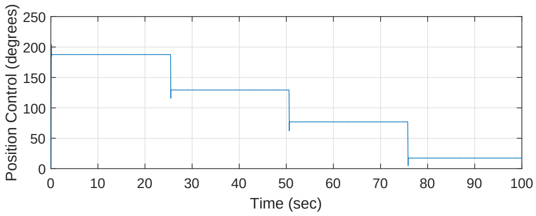


(d) Experimental Magnified control signal for τ=28msec

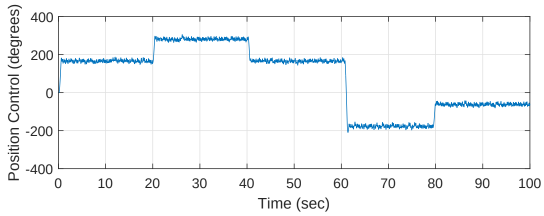
Figure 4.18: Simulation as well as Experimental results of control signal for τ=28msec along with tracking response

delay of $\tau = 10msec$ with $\tau_{sc} = 5msec$ and $\tau_{ca} = 5msec$. The fractional part of total networked delay for sampling interval of $h = 30msec$ is computed to be $\tau' = 0.33$, $\tau'_{sc} = 0.166$ and $\tau'_{ca} = 0.166$ respectively. Figures (4.14(a)) and (4.14(b)) shows the simulated as well as experimental trajectory results of the plant. It can be observed that the position of DC motor is controlled according to variations in the reference inputs without chattering even in the presence of specified network delay. The tracking results are magnified as shown in Figures (4.14(c)) and (4.14(d)) in order to examine the effect of time delay compensation. It can be noticed that in both the cases the fractional part of the delay from sensor to controller is compensated as the position of the motor commence the reference input signal at $5msec$. The same consequence of time delay compensation is observed in sliding variable (4.14(e)) and (4.14(f)) as well as control signal (4.15(a)) and (4.15(b)) respectively. Observing the magnified results (4.14(g)), (4.14(h)), (4.15(c)) and (4.15(d)) it can be noticed that the compensated sliding variable and control signal both are computed from first sampling instants. Thus, the effect of fractional delay from sensor to controller is compensated at the sliding surface and control signal. The proposed algorithm was further examined for $\tau = 18msec$ and $28msec$ respectively. Figures (4.15(e)) to (4.15(h)) shows the results of position control brushless DC motor. The results are carried out under the total networked delay of $\tau = 18msec$ with $\tau_{sc} = 9msec$ and $\tau_{ca} = 9msec$. The fractional part of delay for $h = 30msec$ is computed as $\tau' = 0.6$, $\tau'_{sc} = 0.3$ and $\tau'_{ca} = 0.3$ respectively. The simulated and experimental tracking results of the plant for the specified networked delay are shown in Figures (4.15(e)) and (4.15(f)). It can be noticed that in both the cases the position of DC motor is controlled for all given reference inputs. In order to examine the effect of fractional time delay compensation the same results are magnified in Figures (4.15(g)) and (4.15(h)). It can be noticed that the effect of fractional delay from sensor to controller is nullified as the output follows the reference signal at $t = 9msec$. The same effect of time delay compensation is observed in sliding variable (4.16(a)) and (4.16(b)) as well as control signal (4.16(e)) and (4.16(f))). Observing the magnified results [(4.16(c)), (4.16(d)), (4.16(g)) and (4.16(h))] it can be noticed that in both the cases the sliding variable and control signal are computed from first sampling instants. Thus the effect of fractional part of delay from sensor to controller is compensated at sliding variable and control signal. Figures (4.17(a)) to (4.18(d)) shows the results of position control of brushless DC motor for total networked delay of $\tau = 28msec$ with $\tau_{sc} = 14msec$ and $\tau_{ca} = 14msec$. Considering the sampling interval of $h = 30msec$ the fractional part of delay is computed as $\tau' = 0.933$, $\tau'_{sc} = 0.466$ and $\tau'_{ca} = 0.46$ respectively. Figures (4.17(a)) and (4.17(b)) shows the simulated and experimental tracking results of the system for the specified networked delay. It can be observed that in both the cases, output tracks the reference trajectory without chattering. In order to examine the actual effect of time delay compensation the results are magnified as shown in Figures (4.17(c)) and (4.17(d)). Observing the magnified results it can be concluded that the effect of fractional part of delay from sensor to controller is compensated as output tracks the reference signal at $t = 14msec$. Figures (4.17(e)), (4.17(f)), (4.18(a)) and (4.18(b)) shows the simulated and experimental results of compensated sliding variable and control signal for specified networked delay. It can be noticed from the results that the time delay compensation algorithm works efficiently for large value of τ. The magnified results of the same are shown in Figures (4.17(g)), (4.17(h)), (4.18(c)) and (4.18(d)) which clearly justifies that in both the cases the effect of fractional part of delay from sensor to controller is compensated in the sliding variable as well as control signal.

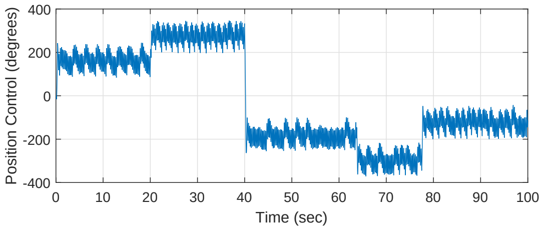
Thus from above results, it can be concluded that the non-switching controller designed using proposed algorithm works efficiently with the network delay range of $10msec$ to $28msec$ in simulated as well as experimental environment. The proposed controller



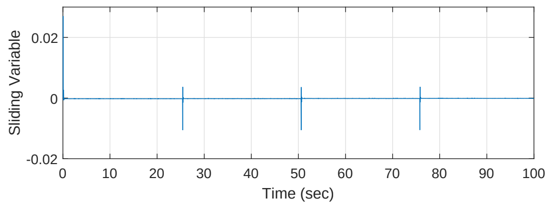
(a) Position control of DC motor using proposed algorithm for τ=12.8msec



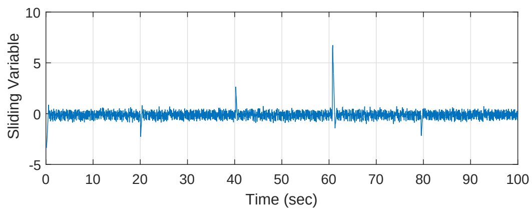
(b) Position control of DC motor using switching based SMC algorithm for τ=12.8msec



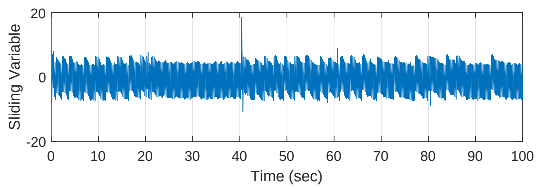
(c) Position control of DC motor using conventional SMC algorithm for τ=12.8msec



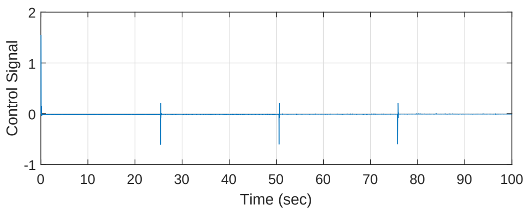
(d) Compensated sliding variable of proposed algorithm algorithm for τ=12.8msec



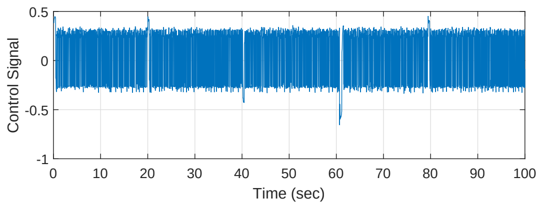
(e) Compensated sliding surface of switching based SMC algorithm for τ=12.8msec



(f) Compensated sliding variable of conventional SMC algorithm for τ=12.8msec



(g) Control signal response of proposed algorithm for τ=12.8msec



(h) Control signal response of switching based SMC algorithm for τ=12.8msec

Figure 4.19: Comparison of proposed algorithm, switching based sliding mode control and conventional sliding mode control for τ=12.8msec

compensates the effect of fractional time delay without chattering for $\psi = 1500$ satisfying (4.4) and shows the stable response satisfying (4.7) in the presence of matched uncertainty.

Comparison of proposed algorithm with conventional sliding mode control

In this section, the experimental results of proposed algorithm are compared with switching sliding mode control using time delay approximation and conventional sliding mode control. The results are compared in terms of tracking response, sliding variable and

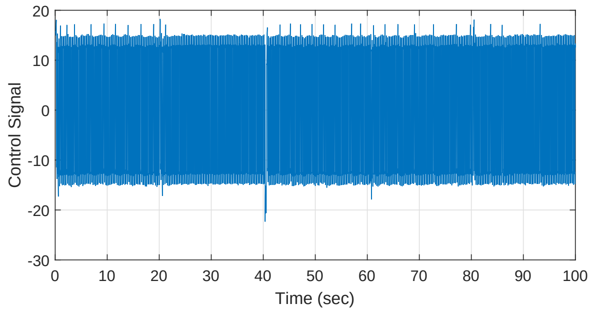


Figure 4.20: Control signal response of coventional SMC algorithm for τ=12.8msec

Algorithm	Comparison Results			
	τ	T_s	$Chattering$	$Response$
$Coventional\ SMC$	12.8msec	Undefined	high	unstable
$Switching\ SMC$	12.8msec	1sec	within QSMB	stable
$Proposed\ method$	12.8msec	0.3sec	negligible within QSMB	stable

Table 4.1: Comparison of Proposed Algorithm, Switching based SMC and Conventional SMC

control signal for total networked delay of $\tau = 10msec$. From the comparative results [Figures (4.19(a) to (4.20)], it can be observed that the conventional sliding mode control becomes unstable for a small delay of $\tau_{sc} = 5msec$ while the sliding mode controller designed using switching algorithm generates the chattering behaviour at the output signal compared to proposed algorithm. Thus, Thiran Approximation proved to be more efficient technique for non-switching based discrete-time sliding mode control. The comparison of proposed algorithm with switching sliding mode control using time delay approximation and conventional sliding mode control are summarized in Table-4.1.

4.4.3 Simulation Results With Real-Time Networks

In previous section the efficacy of the proposed control law is examined in the presence of Brushless DC motor connected through networked medium. It can be observed from simulation results that, the control law proposed using non-switching reaching law provides faster convergence without increasing the amplitude of control signal. The chattering is also negligible compared to switching-type control law. Thus, in this section the efficacy of the proposed non-switching control law is further tested in the presence of real time networks and matched uncertainty. The real time networks are simulated using True Time software which provides wide range of simulated networks such as CAN, Switched Ethernet, Profibus, Profinet, CSMA/CD, Round Robbin etc.. In this work, the simulations are carried out under CAN and Switched Ethernet communication medium as network delays are assumed to be deterministic in nature. Further, the performance of the system is also studied in the presence of packet loss situation.

The following network specifications and parameters are considered for simulations:

$Networked\ Medium$: CAN and Switched Ethernet
$Data\ Rate\ (bits/s)$=80000
$Minimum\ frame\ size\ (bits)$=512 ($CAN$) and 1024 ($Switched\ Ethernet$)
$Loss\ Probability$ = 0 to 0.5
$sampling\ interval\ h$= 0.030 second.
$C_s = [24.5156 \quad 31.6288]$
$|s(k)| \leq +0.2$ to -0.2 with user defined constant $\psi = 1500$.

Consider the continuous-time LTI system as,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dd(t), \qquad (4.23)$$

$$y(t) = Cx(t), \qquad (4.24)$$

where,
$A = \begin{bmatrix} -0.7 & 2 \\ 0 & -1.5 \end{bmatrix}$, $B = \begin{bmatrix} -0.03 \\ -1 \end{bmatrix}$,
$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Discretizing the above system parameters at sampling interval of $h = 30msec$ we get,

$$x(k+1) = Fx(k) + Gu(k-\tau') + d(k), \qquad (4.25)$$

$$y(k) = Cx(k), \qquad (4.26)$$

where,
$F = \begin{bmatrix} 0.9792 & 0.05805 \\ 0 & 0.956 \end{bmatrix}$, $G = \begin{bmatrix} -0.001771 \\ -0.02934 \end{bmatrix}$,

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

CAN as a Network Medium

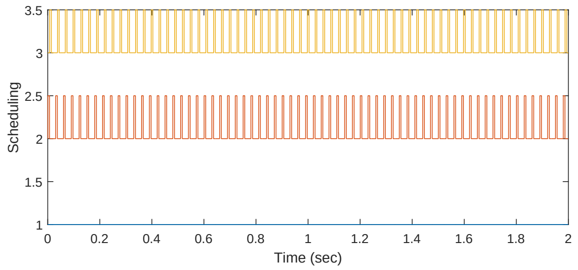


Figure 4.21: Scheduling policies of sensor to actuator with CAN network under ideal condition

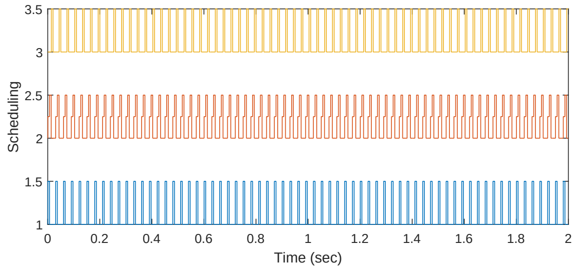


Figure 4.22: Scheduling policies of sensor to actuator with CAN network under traffic conditions

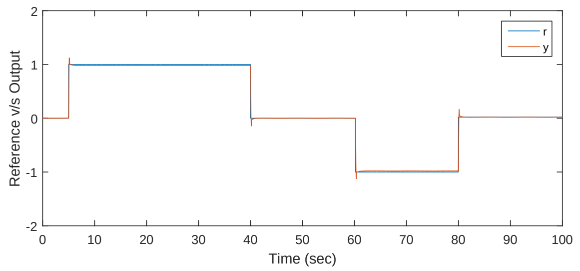


Figure 4.23: Tracking response of system with CAN network under ideal condition

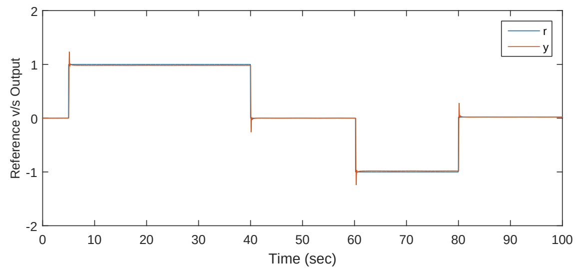


Figure 4.24: Tracking response of system with CAN network under traffic conditions

In this section, the nature of the system with CAN as a networked medium is studied in Figures (4.21) to (4.36). The robustness of the proposed controller is checked

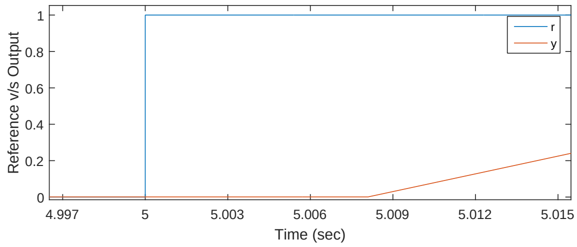


Figure 4.25: Magnified tracking response with CAN network under ideal conditions

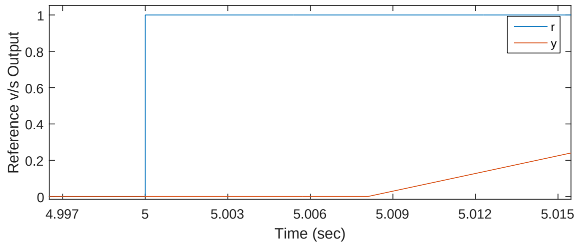


Figure 4.26: Magnified tracking response with CAN network under traffic conditions

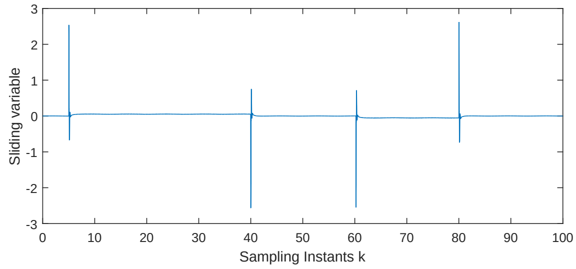


Figure 4.27: Compensated sliding variable $s(k)$

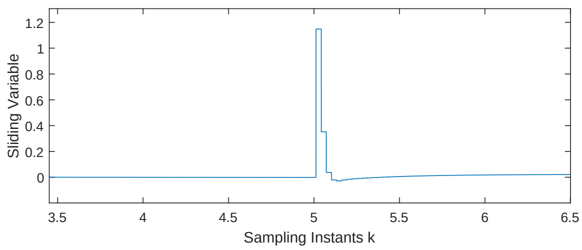


Figure 4.28: Magnified compensated sliding variable $s(k)$

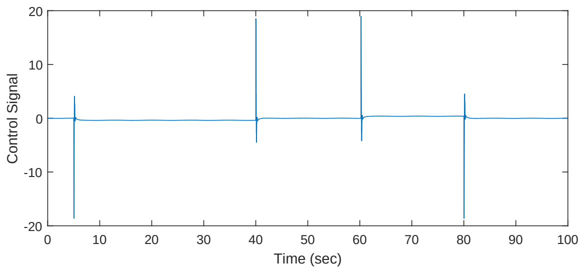


Figure 4.29: Control signal $u(k)$

by applying slowly time varying disturbance at the input side of the system as shown in Figure (4.1). In CAN, it is assumed that the minimum frame size is 512 bits and

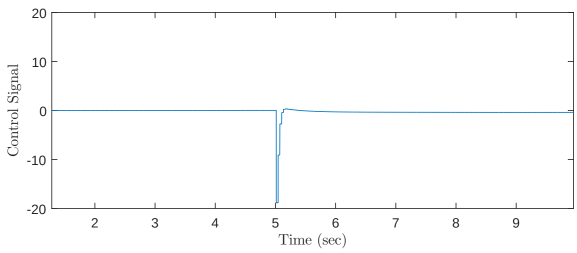


Figure 4.30: Magnified control signal $u(k)$

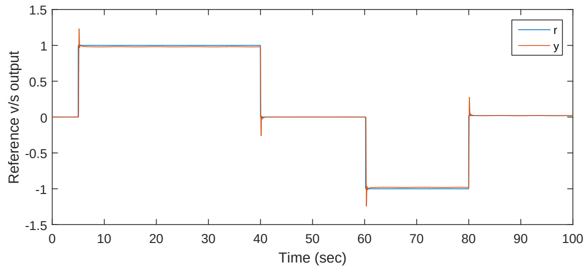


Figure 4.31: Tracking response of system when packet loss is is 10%

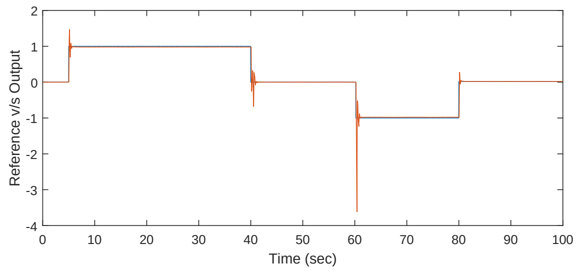


Figure 4.32: Tracking response of system when packet loss is 30%

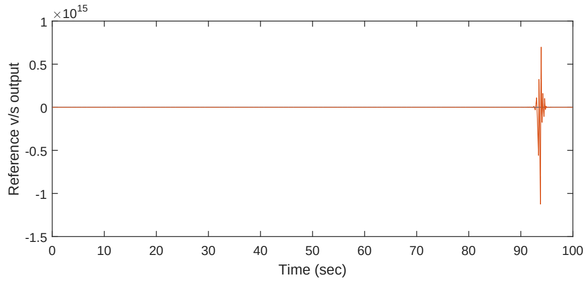


Figure 4.33: Tracking response of system when packet loss is 50%

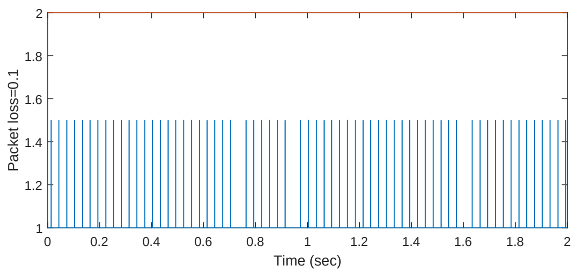


Figure 4.34: Scheduling policy of sensor to controller when packet loss is 10%

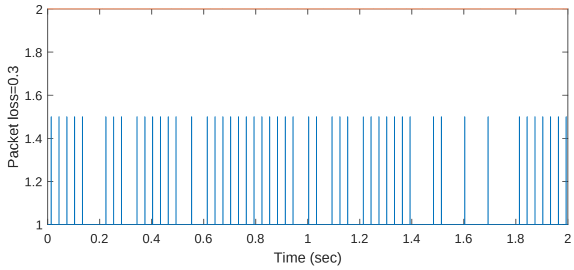


Figure 4.35: Scheduling policy of sensor to controller when packet loss is 30%

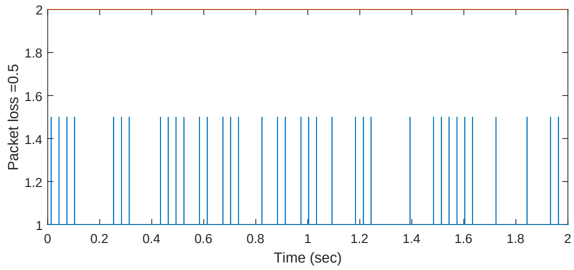


Figure 4.36: Scheduling policy of sensor to controller when packet loss is 50%

data transfer rate is 80000 bits/sec. So, the delay generated in the CAN network to transfer the data packets from sensor to controller is $\tau_{sc} = 6.4msec$ and from controller to actuator is $\tau_{ca} = 6.4msec$. The processing and the computational delays at sensor, controller and actuator are considered as $0.9msec$, $0.5msec$ and $0.5msec$ respectively. Thus the total networked delay generated within the closed loop system is $\tau = 14.7msec$. The fractional part of total network delay is obtained as $\tau' = 0.49$, $\tau'_{sc} = 0.213$ and $\tau'_{ca} = 0.213$ for sampling interval of $h = 30msec$. The scheduling policies of sensor to controller and controller to actuator with network under ideal condition and bandwidth sharing condition are shown in Figures (4.21) and (4.22) respectively. It can be observed that blue samples are indicated as the traffic while yellow and red samples indicate the scheduling policy for sensor to controller and controller to actuator. The trajectory response of the system for the network under ideal condition and traffic condition are shown in Figures (4.23) and (4.24). It can be noticed that under both situations the output tracks the reference trajectory for the specified networked delay. In order to show the precise effect of time delay compensation in CAN network at the output results are magnified as shown in Figures (4.25) and (4.26). It can be noticed that the effect of fractional delay from sensor to controller is compensated as the output tracks the reference input at $t = 8.3msec$. The similar effect of time delay compensation for the network under traffic condition can be observed in sliding variable Figures (4.27) and (4.28) as well as control signal Figures (4.29) and (4.30). Observing the magnified results in Figures (4.28) and (4.30), it can be noticed that the sliding variable and control signal are computed exactly after an interval of $t = 1.4msec$ even in the presence of sensor to controller delay. Apart from time delay compensation, the proposed algorithm was examined under packet loss condition. Figures (4.31), (4.32) and (4.33) shows the results of tracking response under packet loss condition while Figures (4.34), (4.35) and (4.36) shows the instances of packet drop. It can be observed from results that when the packet loss is 50% the system goes to unstable condition. Thus it can be concluded that the system shows the satisfactory response under 30% of packet loss for specified network delay with CAN as a communication medium.

Switched Ethernet as a Network Medium

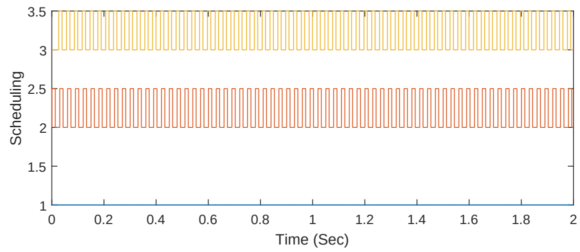


Figure 4.37: Scheduling policies of sensor to actuator of Switched Ethernet network under ideal condition

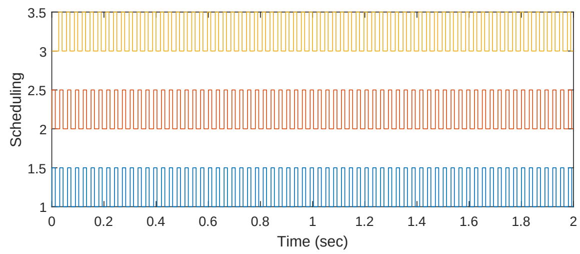


Figure 4.38: Scheduling policies of sensor to actuator of Switched Ethernet network under traffic condition

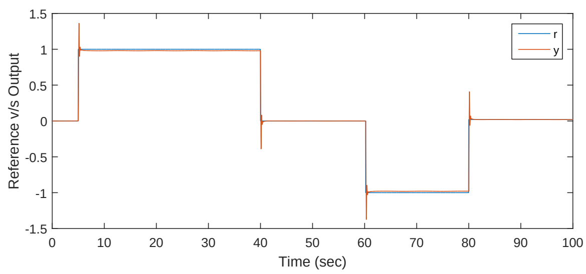


Figure 4.39: Tracking response of the system with Switched Ethernet as networked medium under idle condition.

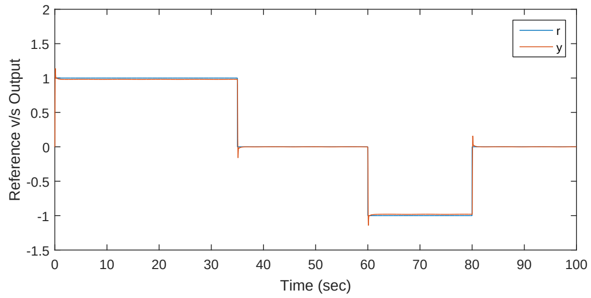


Figure 4.40: Tracking response of the system with Switched Ethernet as networked medium with traffic condition

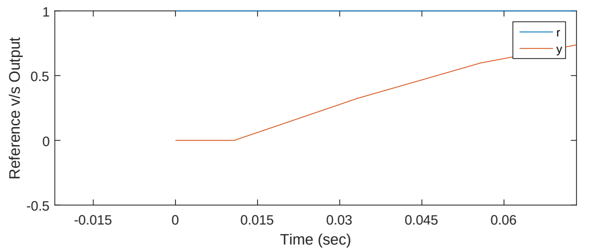


Figure 4.41: Magnified tracking response of the system with network under ideal condition

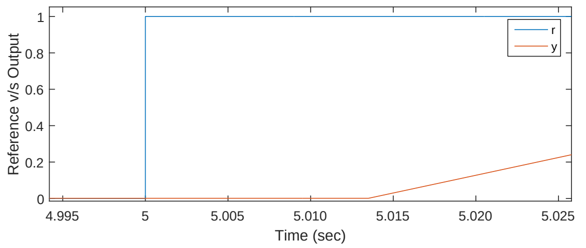


Figure 4.42: Magnified tracking response of the system with network under traffic load

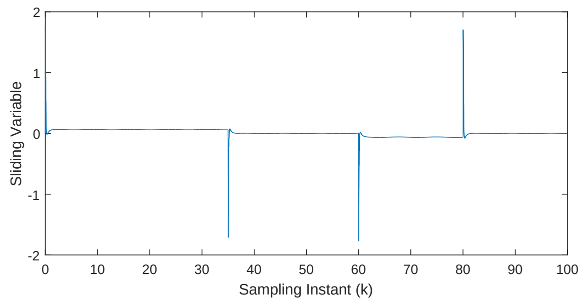


Figure 4.43: Compensated sliding variable $s(k)$

In this segment, the nature of the system is studied in Figures (4.37) to (4.52) for Switched Ethernet type of communication medium considering the delays are deterministic in nature. In Switched Ethernet, the minimum frame size is 1024 bits and data transfer rate is 80000 bits/sec. So, the delay generated within the network to transfer the data packets from sensor to controller is $\tau_{sc} = 12.8msec$ and from controller to actuator

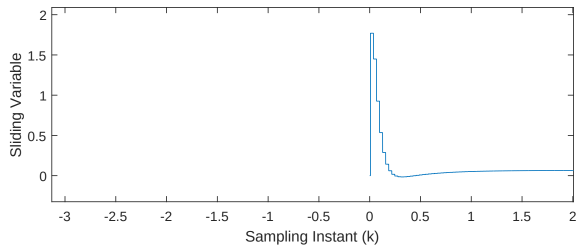


Figure 4.44: Magnified compensated sliding variable $s(k)$

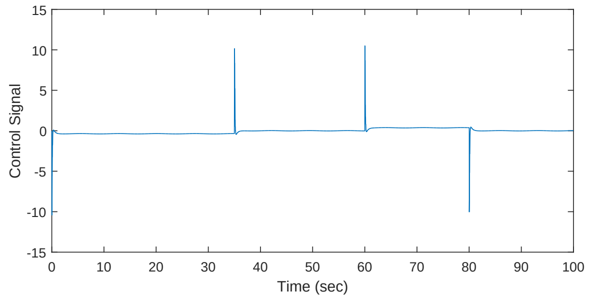


Figure 4.45: Control signal $u(k)$

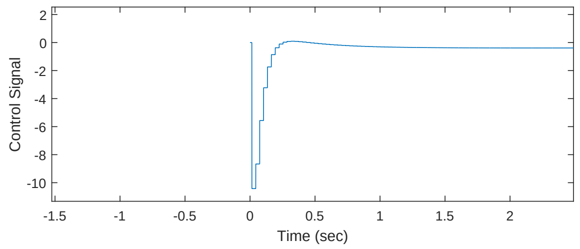


Figure 4.46: Magnified control signal $u(k)$

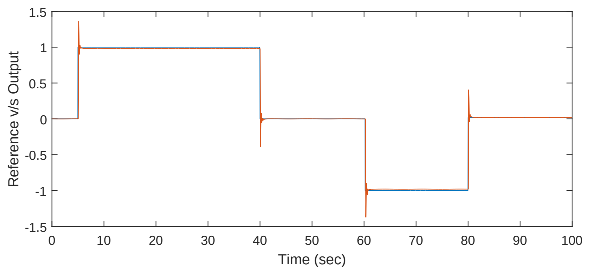


Figure 4.47: Tracking response of system when packet loss is 10%

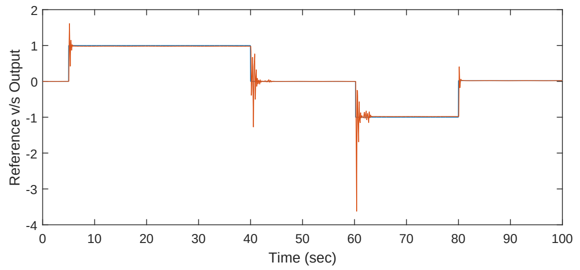


Figure 4.48: Tracking response of system when packet loss is 30%

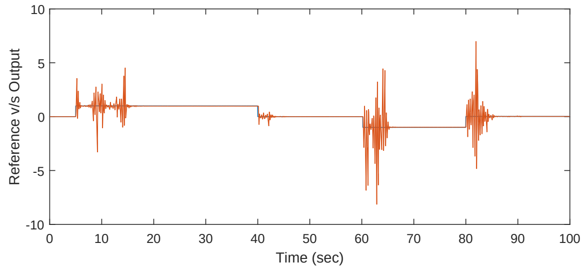


Figure 4.49: Tracking response of system when packet loss is 50%

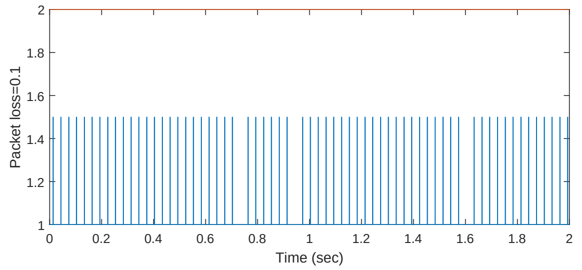


Figure 4.50: Scheduling policy of sensor to controller when packet loss is 10%

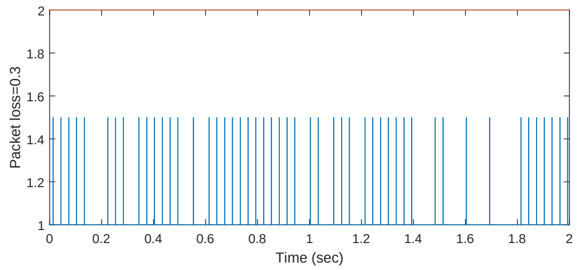


Figure 4.51: Scheduling policy of sensor to controller when packet loss is 30%

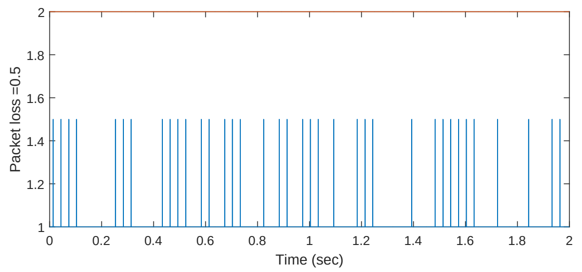


Figure 4.52: Scheduling policy of sensor to controller when packet loss is 50%

is $\tau_{ca} = 12.8msec$. The processing and the computational delays at sensor, controller and actuator are considered as $0.9msec$, $0.5msec$ and $0.5msec$ respectively. Thus the total networked delay within the closed loop system is computed as $\tau = 27.5msec$. The fractional part of total network delay is obtained as $\tau' = 0.91$, $\tau'_{sc} = 0.426$ and $\tau'_{ca} = 0.426$ for sampling interval of $h = 30msec$. The scheduling policies of sensor to controller and controller to actuator with network under ideal condition and bandwidth sharing condition are shown in Figures (4.37) and (4.38) respectively. The trajectory

Algorithm	Comparison Results			
	τ_{CAN}	τ_{Ether}	T_s	$Response$
$Coventional\ SMC$	14.7msec	25.7msec	Undefined	unstable
$Proposed\ method$	14.7msec	25.7msec	1sec	stable

Table 4.2: Comparison of Proposed Algorithm with Conventional SMC in True Time

response of the system for the network under ideal condition and traffic condition are shown in Figures (4.39) and (4.40). It can be noticed that under both the situations the output tracks the reference trajectory for the specified networked delay. In order to show the exact effect of time delay compensation in Switched Ethernet network at the output, results are magnified as shown in Figures (4.41) and (4.42). It can be noticed that the effect of fractional delay from sensor to controller is compensated as the output tracks the reference input at $t = 14.7msec$. The similar effect of time delay compensation for the network under traffic condition can be observed in sliding variable Figures (4.43) and (4.44) as well as control signal Figures (4.45) and (4.46). Observing the magnified results in Figures (4.44) and (4.46), it can be noticed that the sliding variable and control signal are computed exactly after an interval of $t = 1.4msec$ even in the presence of sensor to controller delay. Apart from time delay compensation, the proposed algorithm was examined under packet loss condition. Figures (4.47), (4.48) and (4.49) shows the results of tracking response under packet loss condition while Figures (4.50), (4.51) and (4.52) shows the instances of packet drop. It can be observed from results that when the packet loss is 50% the system goes to unstable condition. Thus, it can be concluded that the system shows the satisfactory response under 30% of packet loss for specified network delay with Switched Ethernet as a communication medium.

Comparison of proposed algorithm with conventional sliding mode control under CAN and Switched Ethernet as a network medium

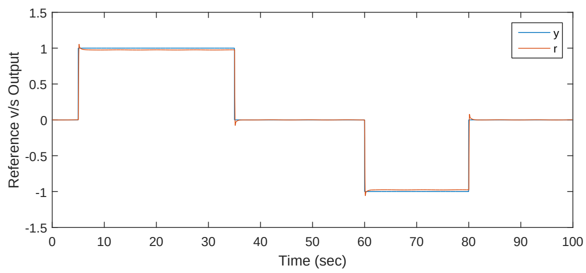


Figure 4.53: Time delay compensation scheme with CAN as a networked medium.

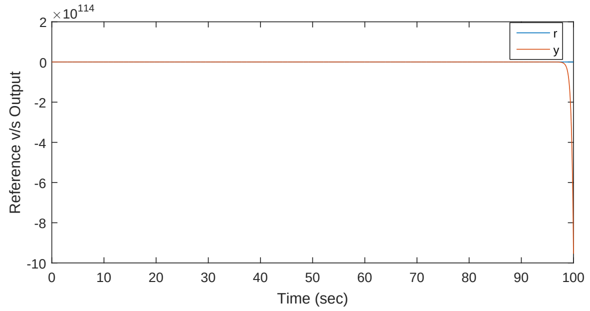


Figure 4.54: Tracking response of conventional SMC with CAN as a communication medium.

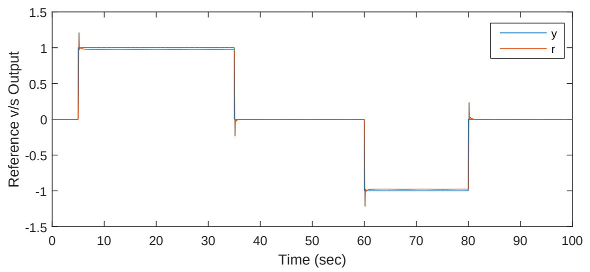


Figure 4.55: Time delay compensation using Switched Ethernet as a communication medium.

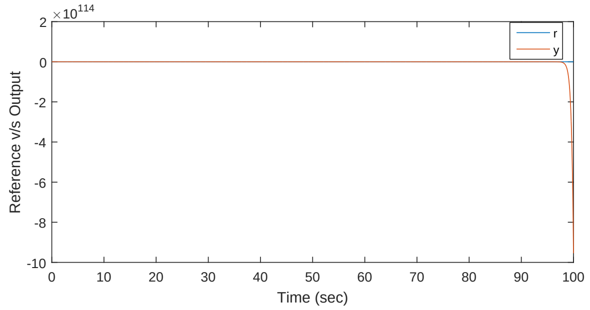


Figure 4.56: Tracking response of conventional SMC with Switched Ethernet as a networked medium.

In this section, the results of proposed algorithm were compared with conventional SMC for CAN and Switch Ethernet communication medium. Figures (4.53), (4.54), (4.55) and (4.56) shows the comparative results of proposed algorithm and conventional SMC for discrete time sliding mode control. It can be observed from comparison that conventional SMC shows unstable response for the specified networked delay. Thus, Thiran approximation proved to be efficient technique in discrete time sliding mode control under packet loss condition and matched uncertainty. The comparison of proposed algorithm and conventional SMC for CAN and Switched Ethernet communication medium are summarized in Table-4.2.

4.5 Conclusion

In this chapter, we explored Thiran's approximation technique for fractional delay compensation in discrete-time domain. Using this approach, non-switching type discrete time sliding mode controller is designed which computes the control actions in the presence of network delay and matched uncertainty. The stability of the closed loop NCS is assured by using Lyapunov approach such that system states remain within the specified band. The effectiveness of the derived algorithms is tested using Illustrative example and brushless DC motor setup with deterministic networked delay and matched uncertainty. The experimental results are compared with non-switching SMC, switching SMC as well as conventional algorithm. The comparative results proved that the fractional delay approximated using Thiran Approximation is most efficient technique as it compensates the fractional network delays in discrete-time domain. Moreover, the simulation results carried out using illustrative example and experimental results carried out in the presence of DC servo motor plant proved that the control algorithm designed using non-switching reaching law is robust and efficient algorithm as it provides the faster convergence with less chattering in discrete-time domain. Further, the efficiency of non-switching controller designed using Thiran Approximation was examined under simulated CAN and Switched Ethernet networked medium using True Time. The results shows that the proposed control algorithm designed using Thiran Approximation compensates the networked delay in the presence of network non-idealities (time delay, packet loss and matched uncertainty).

CHAPTER 5

MULTIRATE OUTPUT FEEDBACK DISCRETE-TIME SLIDING MODE CONTROLLER FOR FRACTIONAL DELAY COMPENSATION

5.1 Introduction

In NCS, if states information is available for measurement then state feedback is the simplest way for designing the SMC controller. In reality for any network based control system most of the states are observable but they are immeasurable. So it is essential to design the SMC controller based on output information which is always measurable. This chapter summarizes the designing of multirate output feedback based discrete-time sliding mode controller in which the control input is computed based on the system outputs and past control signals by taking full advantage of network transmission. A Thiran Approximation technique is used to compensate the networked delays. The sensor to controller delay is compensated at the sliding surface, while sliding mode controller (SMC) is utilized to compute the control sequences. The stability of the closed loop NCSs is derived using Lyapunov approach. Simulations results are given to demonstrate the effectiveness of the proposed approach.

5.2 Problem Formulation

Figure (5.1) represents the schematic diagram of NCS with time delay compensator and multirate output feedback approach. As shown in Figure (5.1), the state information and control information are exchanged between plant and controller through the networks suffering from sensor to controller delay (τ_{sc}) and controller to actuator delay (τ_{ca}). The state information are computed based on multirate output feedback approach. In this approach, the system states are estimated through the output information and error between the computed state and actual state of the system goes to zero once the multirate output sample is available. While, in coventional output feedback the error between the actual states and estimated states decreases asymtotically and approches to zero at infinite time. In multirate output feedback technique, the control inputs and plant output signals are sampled at different sampling intervals. It is assumed that the sensor output measurements are faster than control input signals. The mathematical expression of

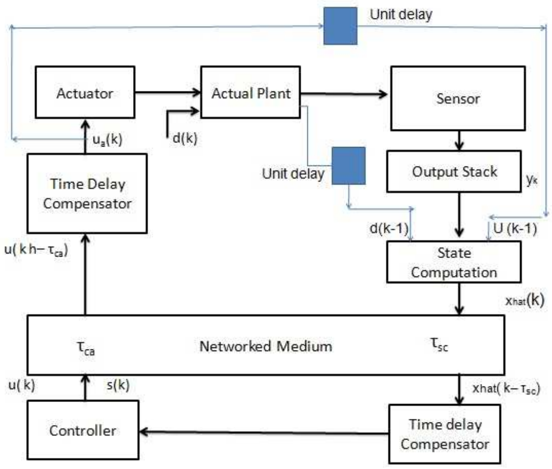


Figure 5.1: Block diagram of NCS with multirate output feedback approach

control input signal sampled at ϕ sampling interval is given by:

$$x((k+1)\phi) = F_{\phi}x(k\phi) + G_{\phi}(u(k\phi) + d(k\phi)), \tag{5.1}$$

$$y(k\phi) = Cx(k\phi). \tag{5.2}$$

Let the output samples be sampled at ζ sampling interval given by,

$$\zeta = \frac{\phi}{\Lambda}, \tag{5.3}$$

where, Λ is the positive interger $\geq$ observability index of the system.
Similarly, the mathematical expression of the sensor output data sampled at ζ sampling interval is given as:

$$x((k+1)\zeta) = F_{\zeta}x(k\zeta) + G_{\zeta}(u(k\zeta) + d(k\zeta)), \tag{5.4}$$

$$y(k\zeta) = Cx(k\zeta). \tag{5.5}$$

$\mathbf{Remark-6}$: The stability of closed loop system is guaranteed, if pair (F_ϕ, G_ϕ) is controllable and pair (F_ζ, C) is observable.

The output of multirate output feedback is expressed in mathematical form as:

$$\hat{x}(k) = H_y y_k + H_u u(k-1) + H_d d(k-1), \tag{5.6}$$

where, y_k represents the output stack, $u(k-1)$ represents the past control input signal, $d(k-1)$ represents the past disturbance signal which are available from measurements.

$H_y = A_\phi (Z_0^T Z_0)^{-1} Z_0^T$, $H_u = B_\phi - H_y X_0$, $H_d = d_\phi - H_y C_d$

$$Z_0 = \begin{bmatrix} C \\ CA_\zeta \\ : \\ : \\ CA_\zeta^{N-1} \end{bmatrix}, X_0 = \begin{bmatrix} 0 \\ CB_\zeta \\ : \\ : \\ C\Sigma_{i=0}^{N-1}(A_\zeta)^i B_\zeta \end{bmatrix},$$

$$C_d = \begin{bmatrix} 0 \\ C_d\zeta \\ : \\ : \\ C\Sigma_{i=0}^{N-1}(A_\zeta)^i d_\zeta \end{bmatrix}, y_k = \begin{bmatrix} y((k-1)\phi) \\ y((k-1)\phi+\zeta) \\ : \\ : \\ y(k\phi-\zeta) \end{bmatrix}.$$

Thus, observing Eqn. (5.6) it can be noticed that the output of multirate depends on past output samples, past control signal and past disturbance signal. Thus, the output of sensor computed based on multirate ouput feedback approach will be applied to the controller through network.

$\mathbf{Remark-7}$: In this chapter nature of the system, total fractional network delays τ', sensor to controller fractional delay τ'_{sc} and controller to actuator fractional delay τ'_{ca} would remain same as mentioned in section (3.3).

$\mathbf{Problem\ Statement}$: The main objective, is to design multirate output feedback based discrete-time sliding mode control law for system (3.3,3.4) in the presence of fractional delay τ'_{sc} and τ'_{ca} satisfying (3.5) and matched uncertainty satisfying (3.7).

The next section describes the mathematical derivation of sliding surface that compensates the effect of fractional delay (τ') in the presence of matched uncertainty $d(k)$ using multirate output feedback approach mentioned in Eqn. (5.6).

5.3 Design of Sliding Surface Using Multirate Output Feedback

For designing DT-SMC, we propose the sliding surface using Thiran Approximation rule and multirate output feedback approach in the form of $\mathbf{Lemma-2}$ as under.

$\mathbf{Lemma-2}$: The sliding surface for the given system (3.3, 3.4) with sensor to controller fractional network delay satisfying assumption (1) and (2) specified in section (3.3) is given as:

$$s(k) = C_s\hat{x}(k) - \alpha C_s\hat{x}(k-1), \tag{5.7}$$

where,

$\alpha = \frac{\hat{\tau}_{sc}}{1+\hat{\tau}_{sc}}$, C_s represents the sliding gain.

$\mathbf{Proof}$: Let the sliding surface with network fractional delay from sensor to controller (τ'_{sc}) is given by:

$$s(k) = C_s\hat{x}(k-\tau'_{sc}), \tag{5.8}$$

where, C_s indicates the sliding gain, $\hat{x}(k-\hat{\tau}_{sc})$ indicates the delayed state vector. The value of sliding gain C_s is computed using discrete LQR method with proper selection of Q and R matrices.

Applying z-transform to Eqn. (5.8) we get:

$$s(z) = C_s\hat{x}(z)z^{-\tau'_{sc}}, \tag{5.9}$$

where, $\tau'_{sc} = \nu = \tau_{sc}/h$.

Considering $\tau'_{sc} \prec 1$ and using Eqn. (3.9) $z^{-\tau'_{sc}}$ can be approximated as,

$$z^{-\tau'_{sc}} = \Sigma_{k=0}^{1}(-1)^k \binom{l}{k} \Pi_{i=0}^{1} \frac{2\tau'_{sc} + i}{2\tau'_{sc} + k + i} z^{-k}. \tag{5.10}$$

The above Eqn. (5.10) can be further expanded as:

$$\begin{aligned} z^{-\tau'_{sc}} = [(-1)^0 \binom{1}{0} \left\{ \frac{2\tau'_{sc}}{2\tau'_{sc}} * \frac{2\tau'_{sc}+1}{2\tau'_{sc}+1} \right\} z^0 + (-1)^1 \binom{1}{1} \\ \left\{ \frac{2\tau'_{sc}}{2\tau'_{sc}+1} * \frac{2\tau'_{sc}+1}{2\tau'_{sc}+2} \right\} z^{-1}]. \end{aligned} \tag{5.11}$$

On simplifying we get,

$$z^{-\tau'_{sc}} = 1 - \alpha z^{-1}. \tag{5.12}$$

where, $\alpha = \frac{\tau'_{sc}}{\tau'_{sc}+1}$.

Substituting Eqn. (5.12) into (5.9),

$$s(z) = [1 - \alpha z^{-1}]\hat{x}(z)C_s. \tag{5.13}$$

Further solving,

$$s(z) = \hat{x}(z)C_s - \alpha z^{-1}\hat{x}(z)C_s. \tag{5.14}$$

Appling inverse z-transform we have,

$$s(k) = C_s\hat{x}(k) - \alpha C_s\hat{x}(k-1). \tag{5.15}$$

This completes the $\mathbf{proof}$.

Observing Eqn. (5.15) it can be noticed that the sliding surface $s(k)$ depends past output samples, past control signal, past disturbance signal and parameter $'\alpha'$ approximated through thiran approximation rule. Thus, the effect of sensor to controller fractional delay τ'_{sc} in the sliding surface at each sampling instants h is compensated through proposed technique.

Now, we are ready to propose multirate output feedback based discrete-time sliding mode control law using the sliding surface (5.7).

5.4 Design of Discrete-Time Networked Sliding Mode Control (DNSMC) Using Multirate Output Feedback

In this section, discrete-time sliding mode control law is derived for deterministic network fractional delays (τ'_{sc} and τ'_{ca}) along with its stability using sliding surface (5.7) represented in form of $\mathbf{Theorem-3}$.

$\mathbf{Theorem-3}$: The discrete-time sliding surface (5.7) is reached in a finite time in the presence of networked delays satisfying (3.5) and matched uncertainty (3.7) provided the control law is designed as:

$$u(k) = -(C_sG)^{-1}[H_{dy}y_k + H_{ud}u(k-1) + H_{dd}d(k-1) - Js(k) - d_s(k) \\ +d_1] - d(k). \tag{5.16}$$

where,
$H = (C_sF)$, $I = \alpha C_s$, $J = [1 - q(s_c(k))]$, $(H - I)H_y = H_{dy}$ $(H - I)H_u = H_{ud}$, $(H - I)H_d = H_{dd}$.

$\mathbf{Proof}$: The reaching law proposed in [Bartoszewicz and Lesniewski (2014)], is used to derive the control law since it provides faster convergence. The reaching law in the presence of sensor to controller fractional delay (τ'_{sc}) is given by:

$$s[(k+1)] = \{1 - q[s(k)]\}s(k) - d(k) + d_1, \tag{5.17}$$

where,
$\{q[s(k)]\} = \frac{\psi}{\psi+|s(k)|}$, $d(k)$ represents the disturbance,
$d_1 = \frac{d_u+d_l}{2}$, mean value of $d(k)$, $d_2 = \frac{d_u-d_l}{2}$, deviated value of $d(k)$ and ψ is the designer's constant satisfying,

$$\psi \geq d_2. \tag{5.18}$$

The reaching law in Eqn. (5.17) contains the disturbance term which is applied through the network. Thus, the compensated disturbance $d_s(k)$ is given as:

$$d_s(k) = d(k) - \alpha d(k-1). \tag{5.19}$$

Using Eqn. (5.7), Eqn. (5.17) can be rewritten as

$$\hat{x}(k+1)C_s - \alpha C_s \hat{x}(k) = [1 - q(s(k))]s(k) - d_s(k) + d_1. \tag{5.20}$$

Substituting the value of $\hat{x}(k+1)$ in Eqn. (5.20),

$$C_s[F\hat{x}(k) + G(u(k) + d(k))] - \alpha C_s \hat{x}(k) = [1 - q(s(k))]s(k) - d_s(k) + d_1. \tag{5.21}$$

Further simplification gives,

$$C_s F\hat{x}(k) + C_s G(u(k) + d(k)) - \alpha C_s \hat{x}(k) = [1 - q(s(k))]s(k) - d_s(k) + d_1. \tag{5.22}$$

The above Eqn. (5.22) further can be expressed in the terms of control law as,

$$\begin{aligned} u(k) = -(C_s G)^{-1}[H_{dy} y_k + H_{ud} u(k-1) + H_{dd} d(k-1) - Js(k) - d_s(k) \\ + d_1] - d(k). \end{aligned} \tag{5.23}$$

This completes the **proof**.

Observing the Eqn. (5.23) it can be noticed that the control law is computed based on the information available in the output stack y_k and not the state information. Thus, using sliding surface (5.7) and control law (5.16) the stability of the closed loop NCS is derived such that the system states will remain within the band in the presence of network fractional delay (τ') and matched uncertainty $d(k)$.

5.4.1 Stability

The system states in (3.3, 3.4) will slides on the sliding surface (5.7) and maintain on it with the controller designed in (5.23) under network fractional delay (τ') satisfying

(3.5) and matched uncertainty (3.7) such that for any $\psi \geq d_2$ and $\gamma \prec 0$ the following condition should hold true:

$$0 \preceq \kappa \prec s^T(k)s(k). \tag{5.24}$$

Proof : Consider the sliding surface (5.7) as,

$$s(k+1) = C_s\hat{x}(k+1) - \alpha C_s\hat{x}(k). \tag{5.25}$$

Let the Lyapunov function be given by,

$$V_s(k) = s^T(k)s(k). \tag{5.26}$$

Taking forward difference of the above Eqn. (5.26),

$$\Delta V_s(k) = s^T(k+1)s(k+1) - s^T(k)s(k). \tag{5.27}$$

Substituting the value of $s(k+1)$ from Eq. (5.7) we get,

$$\begin{aligned}\Delta V_s(k) = [C_s\hat{x}(k+1) - \alpha C_s\hat{x}(k)]^T[C_s\hat{x}(k+1) - \alpha C_s\hat{x}(k)] \\ -s^T(k)s(k).\end{aligned} \tag{5.28}$$

Substituting the value of $\hat{x}(k+1)$ we get,

$$\begin{aligned}\Delta V_s(k) = [C_s[F\hat{x}(k) + G(u(k) + d(k))] - \alpha C_s\hat{x}(k)]^T[C_s[F\hat{x}(k) + G(u(k) \\ +d(k))] - \alpha C_s\hat{x}(k)] - s^T(k)s(k).\end{aligned} \tag{5.29}$$

Substituting the value of $u(k)$ and further solving it we have,

$$\begin{aligned}\Delta V_s(k) = [[1 - q(s(k))]s(k) - d_c(k) + d_1]^T * [[1 - q(s(k))]s(k) - d_c(k) \\ +d_1] - s^T(k)s(k).\end{aligned} \tag{5.30}$$

It can be seen that the term $[[1 - q(s(k))]s(k) - d_c(k) + d_1]$ contains the disturbance term in $\Delta V_s(k)$. Let $\kappa = [[1-q(s(k))]s(k)-d_c(k)+d_1]^T*[[1-q(s(k))]s(k)-d_c(k)+d_1]$

Then we have,

$$\Delta V_s(k) = \kappa - s^T(k)s(k). \tag{5.31}$$

If κ is closed tuned to zero by appropriately selecting the parameter ψ, then $s^T(k)s(k)$ will be larger than κ. Therefore, for any constant parameter γ, we have $\Delta V_s(k) \prec -\gamma s^T(k)s(k)$ which guarantees the convergence of $\Delta V_s(k)$.
This completes the $\mathbf{proof}$.

5.5 Results and Discussion

In this section the efficacy of the designed control algorithm based on MROF approach is validated in the presence of fractional network delay and matched uncertainty applied at the input channel of the system. The simulation results are carried out using illustrative example [Wu and Chen (2007)] in MATLAB enviornment considering different fractional delays.

5.5.1 Simulation Results

Consider the continuous-time LTI system as,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau) + Dd(t), && (5.32)\\ y(t) &= Cx(t), && (5.33) \end{aligned}$$

where,
$A = \begin{bmatrix} -0.7 & 2 \\ 0 & -1.5 \end{bmatrix}$, $B = \begin{bmatrix} -0.03 \\ -1 \end{bmatrix}$,
$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Discretizing the above system with sampling interval of $h = 30msec$ we get,

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k-\tau') + d(k), && (5.34)\\ y(k) &= Cx(k), && (5.35) \end{aligned}$$

where,

$$F = \begin{bmatrix} 0.9792 & 0.05805 \\ 0 & 0.956 \end{bmatrix}, G = \begin{bmatrix} -0.001771 \\ -0.02934 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

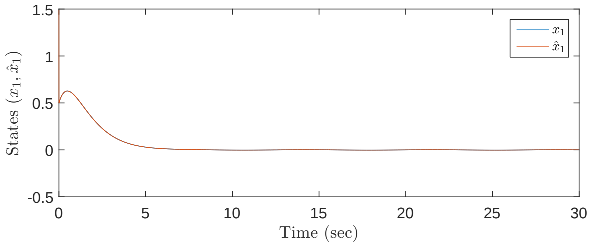


Figure 5.2: Actual state x_1 and estimated state $\hat{x}_1$ with initial condition x_1=0.5 for $\tau = 12.8msec$

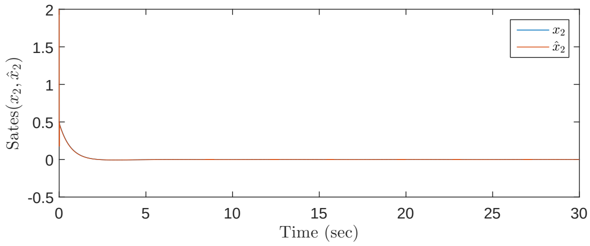


Figure 5.3: Actual state x_2 and estimated state $\hat{x}_2$ with initial condition x_2=0.5 for $\tau = 12.8msec$

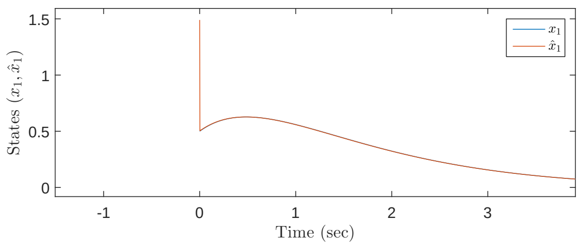


Figure 5.4: Magnified result of actual state x_1 and estimated state $\hat{x}_1$ with initial condition x_1=0.5 for $\tau = 12.8msec$

Figures (5.2) to (5.18) shows the nature of system using multirate output feedback approach under networked environment. The robustness of the system is validated by applying slow time varying disturbance applied at the input side of channel. The networked delay is considered as the time required for the data packets to travel from sensor to controller and controller to actuator. The sliding gain parameter is calculated using discrete LQR method which comes out to be $C_s = [-1.77 \quad -2.766]$ for

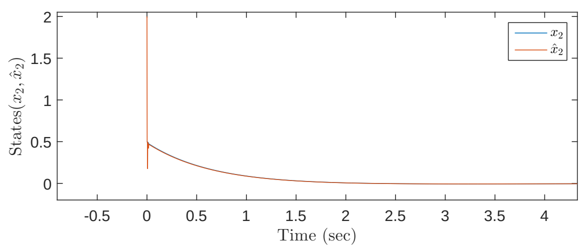


Figure 5.5: Magnified result of actual state x_2 and estimated state $\hat{x}_2$ with initial condition x_2=0.5 for $\tau = 12.8msec$

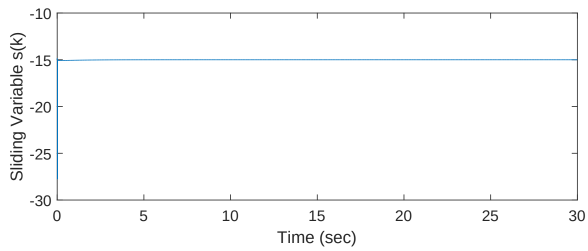


Figure 5.6: Sliding surface $s(k)$ for $\tau = 12.8msec$

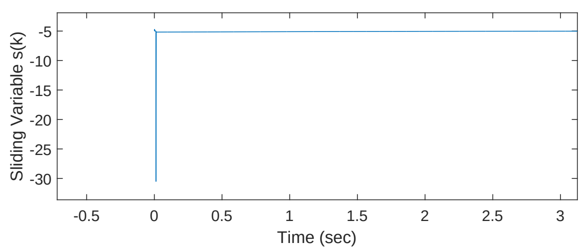


Figure 5.7: Magnified sliding surface $s(k)$ for $\tau = 12.8msec$

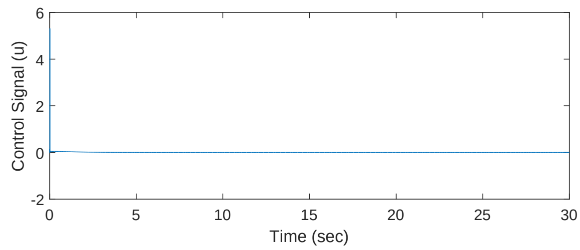


Figure 5.8: Control signal $u(k)$ for $\tau = 12.8msec$

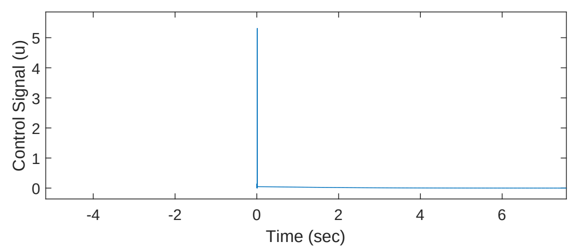


Figure 5.9: Magnified control signal $u(k)$ for $\tau = 12.8msec$

$Q = diag(1000, 1000)$ and $R = 1$. The sliding band $|s(k)|$ remains same as that of previous cases. The initial condition of the system in both the cases are considered

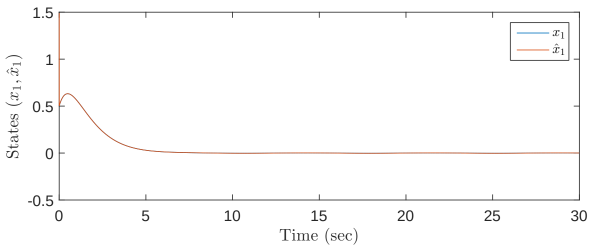


Figure 5.10: Actual state x_1 and estimated state $\hat{x}_1$ with initial condition x_1=0.5 for $\tau = 25.6msec$

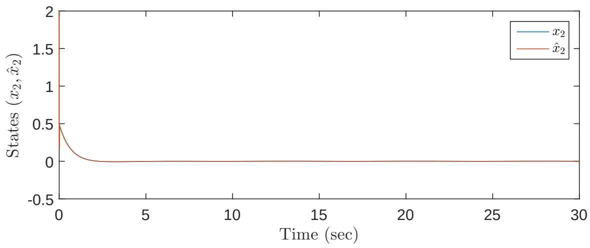


Figure 5.11: Actual state x_2 and estimated state $\hat{x}_2$ with initial condition x_2=0.5 for $\tau = 25.6msec$

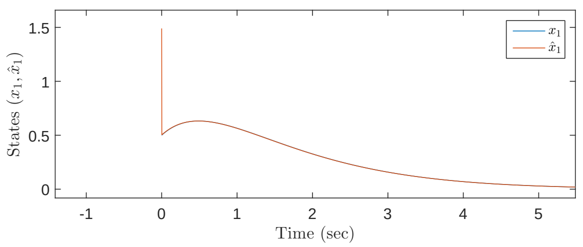


Figure 5.12: Magnified result of actual state x_1 and estimated state $\hat{x}_1$ with initial condition x_1=0.5 for $\tau = 25.6msec$

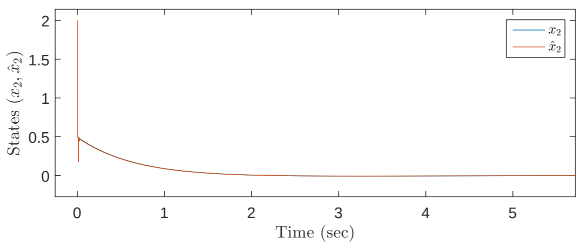


Figure 5.13: Magnified result of actual state x_2 and estimated state $\hat{x}_2$ with initial condition x_2=0.5 for $\tau = 25.6msec$

as $[x_1 \quad x_2] = [0.5 \quad 0.5]^T$. The sensor output signal is sampled at $\zeta = 2msec$ and the control input signal is sampled at $\phi = 6msec$ considering $\Lambda = 3$. The efficacy of the proposed algorithm is checked under networked delay of $\tau = 12.8msec$ and $\tau = 25.6msec$ respectively.

Figures (5.2) to (5.9) show the regulatory response of state variables, sliding variable and control signal for $\tau = 12.8msec$ with $\tau_{sc} = 6.4msec$ and $\tau_{ca} = 6.4msec$. Since

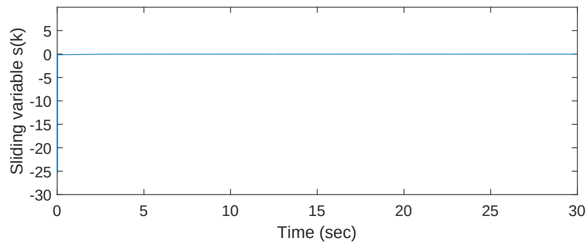


Figure 5.14: Sliding surface $s(k)$ for $\tau = 25.6msec$

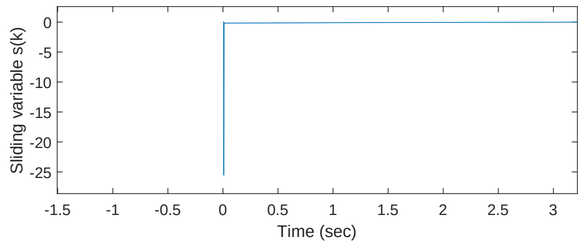


Figure 5.15: Magnified sliding surface $s(k)$ for $\tau = 25.6msec$

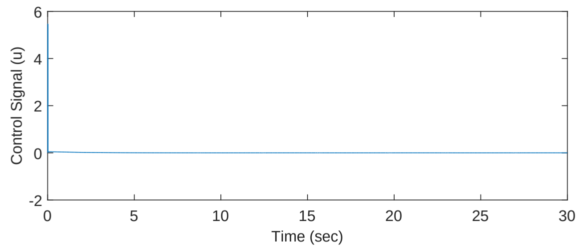


Figure 5.16: Control Signal $u(k)$ for $\tau = 25.6msec$

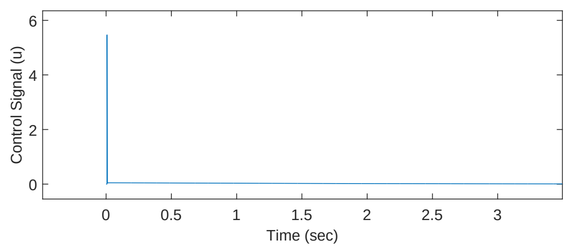


Figure 5.17: Magnified control signal $u(k)$ for $\tau = 25.6msec$

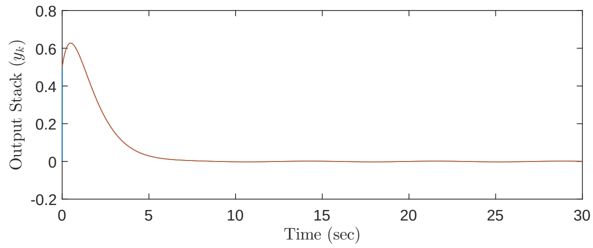


Figure 5.18: Output stack y_k

the system is discretized at $h = 30msec$, the fractional part of delay is computed to be $\tau' = 0.426$, $\tau'_{sc} = 0.213$ and $\tau'_{ca} = 0.213$ respectively. Figures (5.2) and (5.3) show the regulatory response of the system states for the specified networked delay. It can be observed that the actual states (x_1, x_2) and estimated states $(\hat{x}_1, \hat{x}_2)$ both converges to zero from their given initial condition. In order to prove the effect of multirate output feedback and time delay compensation the results are magnified as shown in Figures (5.4) and (5.5) respectively. It can be noticed that in both cases the estimated state variables follows the actual state variable at $\zeta = 2msec$ which means the error becomes zero once the multirate output sample is available. Apart from these, both the states are computed from first sampling instants. Thus, the effect of fractional time delay from sensor to controller and controller to actuator is compensated. The similar effect of compensation is observed in sliding variable and control signal results shown in Figures (5.6) to (5.9). Observing the magnified results shown in Figures (5.7) and (5.9) it can be noticed that the sliding variable and control signal both are computed from first sampling instants even in the presence of sensor to controller fractional delay.

Figures (5.10) to (5.17) show the response of the system in terms of state variables, sliding variable and control signal for $\tau = 25.6msec$ with $\tau_{sc} = 12.8msec$ and $\tau_{ca} = 12.8msec$. The fractional part of delay is computed to be $\tau' = 0.8533$, ${\tau'}_{sc} = 0.426$ and ${\tau'}_{ca} = 0.426$ for $h = 30msec$. Figures (5.10) and (5.11) show the regulatory response of the system state variables for specified networked delay. It can be noticed that actual states (x_1, x_2) and estimated states $(\hat{x}_1, \hat{x}_2)$ both converges to zero from their given initial condition. The magnified regulatory response of the same is shown in Figures (5.12) and (5.13). It can be observed that both states variables start converging to zero at first sampling instant and the estimated states follows the actual state variables at $\zeta = 2msec$. Thus, the effect of network delay from sensor to controller and controller to actuator is nullified at the output. The same effect is observed in sliding variable Figure (5.14) as well as control signal Figure (5.16). Observing the magnified results shown in Figures (5.15) and (5.17), it can be concluded that the effect of fractional delay from sensor to controller is compensated through Thiran Approximation. Figure (5.18) shows the result of output stack for both the cases.

Thus, from above results it can be noticed that the proposed control algorithm works efficiently for the networked delay of $12.8msec$ to $25.6msec$. The sliding surface and control algorithm derived using multirate output feedback approach with Thiran Approximation takes care of sensor to controller fractional delay and controller to actuator fractional delay in the presence of matched uncertainty.

5.6 Conclusion

In this chapter, a new idea of compensating the fractional delay in forward as well as feedback channel is introduced using the concept of multirate output feedback approach. The sensor to controller fractional delay is compensated using Thiran Approximation at the sliding surface while controller to actuator fractional delay is compensated at actuator end. Using this novel approach a multirate output feedback discrete-time networked sliding mode control law is derived that compute the control sequences in the presence of network fractional delay and matched uncertainty. The main advantage of using multirate output feedback approach is that the system states are computed based on the output information available and the error between computed as well as estimated state variables becomes zero exactly after one sampling instant. Stability of the closed loop NCS is ensured using Lyapunov approach such that system states would remain within the band under network non-idealities. The simulation results carried out using proposed technique shows the enhanced response and compensates the effect of fractional delay accurately in discrete-time domain. The proposed algorithm is validated for deterministic type of network fractional delay.

CHAPTER 6

DISCRETE-TIME SLIDING MODE CONTROLLER FOR RANDOM FRACTIONAL DELAYS AND PACKET LOSS

6.1 Introduction

In Networked Control System the behaviour of network delays generally depends on the characteristics of communication medium as well as occupancy of channel by different elements. When large number of sensors, controllers and actuators share their information through the common comunication medium then the network delays and packet losses are random in nature. In this chapter, a novel approach is presented for designing discrete-time sliding mode controller by treating random fractional delay and packet loss seperately. The fractional delay that occurs within sampling period while transmitted from sensor to controller and controller to actuator channel are modelled using Poisson's distribution function and are approximated using Thiran's delay approximation technique for designing the discrete-time sliding mode controller. The packet loss that occur in communication channel between sensor to controller and controller to actuator are treated with Bernoulli's distribution function and compensated at controller end as well as actuator end. Further, Lyapunov approach is used to determine the stability of closed loop NCSs with proposed discrete-time SMC controller. The feasibility and efficiency of the proposed control methodology is outlined through simulation and experimental results which shows a significant response even in the presence of random fractional delay, packets loss and matched uncertainities.

6.2 Problem Formulation

The block diagram of NCS with communication networked medium and packet loss situation is shown in Figure (6.1). The state information as well as control signal are transmitted to the controller and actuator through the network medium. During transmission, the state information and control signal experience time delay from sensor to controller channel and controller to actuator channel respectively. These delays are broadly defined as the time required for the data packets to travel within the network. While transmission, if data packets takes longer duration of time to travel within the network than sampling interval it is called as packet loss. This situation mainly arises due to network congestion, node competition or heavy traffic in the network.

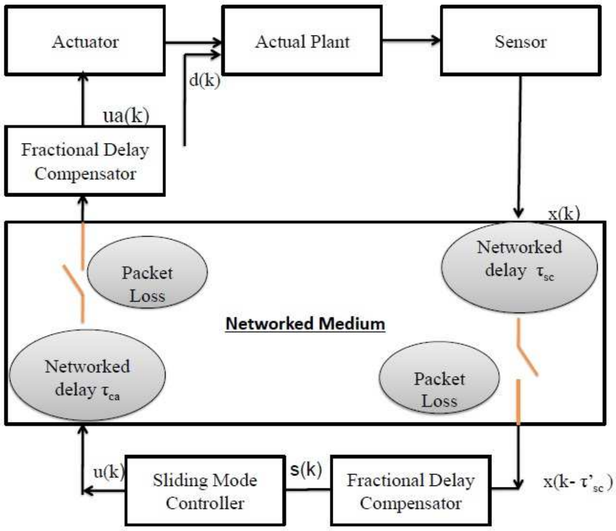


Figure 6.1: Block diagram of NCS with fractional delay compensation and packet loss

Consider the linear time invariant SISO system with random network delay in continuous-time domain as:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_r) + Dd(t), \tag{6.1}$$

$$y(t) = Cx(t), \tag{6.2}$$

where $x \in R^n$ is system state vector, $u \in R^m$ is control input, $y \in R^p$ is system output, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{n\times m}$ are the matrices of appropriate dimensions, $d(t)$ is the matched bounded disturbance with $|d(t)| \leq d_{max}$ and τ_r is the total random network delay in continuous-time domain.

The discrete form of system (6.1) and (6.2) is:

$$x(k+1) = Fx(k) + Gu(k-\hat{\tau}) + d(k), \quad (6.3)$$
$$y(k) = Cx(k), \quad (6.4)$$

where $F = e^{Ah}$, $G = \int_0^h e^{At}Bdt$, $d(k) = \int_0^h e^{At}Dd((k+1)h-t)dt \in O(h)$. Since $|d(t)| \leq d_{max}$, it can be inferred that $d(k)$ is also bounded and $O(h)$ [Mehta and Bandyopadhyay (2016)]. For simplicity, it is assumed that $d(k)$ is slowly varying and remain constant over the interval $kh \leq t \leq (k+1)h$ [Mehta and Bandyopadhyay (2016)].
The total random fractional delay ($\hat{\tau}$) occurring within the network denoted as,

$$\hat{\tau} = \frac{\tau_r}{h},$$

where h is the sampling interval.
$\mathbf{Remark-8}$: In this work, the non-integer form of random fractional delay $\hat{\tau}$ is considered instead of integer form in discrete-time domain in order to compensate the precise effect of random network delay occuring at each sampling instants.
$\mathbf{Assumption-3}$: Network induced fractional delays are random in nature and therefore satisfying

$$\hat{\tau}_l \leq \hat{\tau} \leq \hat{\tau}_u, \quad (6.5)$$

where $\hat{\tau}_l$ and $\hat{\tau}_u$ indicates the lower and upper bound of total random fractional network delays.
The total random network induced fractional delay is the combination of sensor to controller fractional delay ($\hat{\tau}_{sc}$) and controller to actuator fractional delay ($\hat{\tau}_{ca}$) having random behaviour which is given as,

$$\hat{\tau} = \hat{\tau}_{sc} + \hat{\tau}_{ca}, \quad (6.6)$$

where, $\hat{\tau}_{sc} = \frac{\tau_{rsc}}{h}$ and $\hat{\tau}_{ca} = \frac{\tau_{rca}}{h}$.

$\mathbf{Assumption-4}$: In this work, it is assumed that only single packet loss occurs. The assumption is justifying as the packet time delay more than sampling period is considered as dropped or lost packet.

Problem Statement: To design robust non-switching discrete-time sliding mode controller for the system (6.3,6.4) in the presence of random fractional delays $\hat{\tau}_{sc}$ and $\hat{\tau}_{ca}$, matched uncertainty and packet loss situation satisfying condition (6.5), (3.7) and assumption (4).

The sliding mode controller involves the sliding surface design that steers the system states towards the surface and control law that computes the control sequences.

The next section proposes the design of sliding surface that compensates the effect of random fractional delay occuring between sensor to controller in NCSs. It also discusses about the modelling of random fractional delay and packet loss.

6.3 Sliding Surface with Random Fractional Delay and Packet Loss

The effect of random fractional delay in discrete-time domain due to presence of networked medium is compensated in sliding surface presented in $\mathbf{Lemma-3}$.

$\mathbf{Lemma-3}$: The sliding surface $s(k)$ for a random fractional delay ($\hat{\tau}_{sc}$), satisfying conditions (6.5) and (3.7) with packet loss for the system (6.3,6.4) is given as:

$$s(k) = (1-\bar{\alpha})x'_c(k) + \bar{\alpha}x'_c(k-1), \tag{6.7}$$

where,
$x'_c(k) = C_s x(k) - \varsigma C_s x(k-1)$, $x'_c(k-1) = C_s x(k-1) - \varsigma C_s x(k-2)$,$\varsigma = \frac{\hat{\tau}_{sc}}{\hat{\tau}_{sc}+1}$, C_s is the sliding gain and $\bar{\alpha}$ is the probability of the data packet lost.

Proof: In discrete-time domain there are various algorithms for modelling the random variables such as Bernoulli's distribution, Geometric distribution, Poisson's distribution, Probability distribution, Binomial distribution and Pascal distribution. Among these algorithms Bernoulli's distribution and Probability distribution are two widely used algorithms for mathematical modelling of integer type of network delays in discrete-time domain. Since, the sensor to controller delay ($\hat{\tau}_{sc}$) is random and fractional in nature, Poisson's distribution is the most suitable approach in discrete-time domain. Poisson's distribution is used to model the random variables of smaller values on the basis of number of events occurred over the specified interval of time. The occurrence of the events is based on the number of trials required to generate the event. So, using this approach the random fractional delay is modelled by assuming that at each sampling instants an event takes place such that the networked delay might be lesser or higher than sampling interval.

Thus, the communicated state variable over the network having random fractional delay from sensor to controller is given as:

$$x_c(k) = x(k - \hat{\tau}_{sc}), \tag{6.8}$$

where, $\hat{\tau}_{sc}$ is the fractional form of sensor to controller delay in discrete-time domain that takes the values in a finite set, that is, $\{\hat{\tau}_{sc}\}\epsilon\{d_1, d_2, ..., d_q\}$. Thus, the sensor to controller fractional delay $\{\hat{\tau}_{sc}\}$ can be modelled using Poisson's distribution with probabilities given by,

$$Pr\{\hat{\tau}_{sc} = d_v\} = E\{d_v\} = \beta_v, v = 1, 2, ..., q. \tag{6.9}$$

where, β_v is the positive scalar and $\Sigma_{v=0}^{q}\beta_v = 1$, $E\{d_v\}$ is the expectation of the stochastic variable d_v. The mathematical representation of β_v with poisson's distribution is given by:

$$\beta_v = \frac{\lambda^w e^{-\lambda}}{w!}; w = 0, 1, 2, 3, ... \tag{6.10}$$

where, w indicates the number of trials, λ denotes average number of events per interval and e denotes the Euler's number.

Applying z-Transform to Eqn. (6.8) we get,

$$x_c(z) = x(z)z^{-\hat{\tau}_{sc}}, \tag{6.11}$$

where, $\hat{\tau}_{sc} = \nu = \frac{\tau_{rsc}}{h}$.

Using Eqn. (3.9), $z^{-\hat{\tau}_{sc}}$ can be approximated as,

$$z^{-\hat{\tau}_{sc}} = \Sigma_{k=0}^{1}(-1)^k \binom{l}{k} \Pi_{i=0}^{1} \frac{2\hat{\tau}_{sc} + i}{2\hat{\tau}_{sc} + k + i} z^{-k}, \tag{6.12}$$

The above Eqn. (6.12) can be further expanded as:

$$\begin{aligned} z^{-\hat{\tau}_{sc}} = [(-1)^0 \binom{1}{0} \left\{ \frac{2\hat{\tau}_{sc}}{2\hat{\tau}_{sc}} * \frac{2\hat{\tau}_{sc} + 1}{2\hat{\tau}_{sc} + 1} \right\} z^0 + (-1)^1 \binom{1}{1} \\ \left\{ \frac{2\hat{\tau}_{sc}}{2\hat{\tau}_{sc} + 1} * \frac{2\hat{\tau}_{sc} + 1}{2\hat{\tau}_{sc} + 2} \right\} z^{-1}], \end{aligned} \tag{6.13}$$

On simplifying we get,

$$z^{-\hat{\tau}_{sc}} = 1 - \varsigma z^{-1}, \tag{6.14}$$

where, $\varsigma = \frac{\hat{\tau}_{sc}}{\hat{\tau}_{sc}+1}$ and $\hat{\tau}_{sc}$ is the random fractional delay defined in Eqns. (6.9) and (6.10) respectively.

Substituting Eqn. (6.14) into (6.11) we have,

$$x_c(z) = x(z)[1 - \varsigma z^{-1}], \tag{6.15}$$

Further expanding it we obtain,

$$x_c(z) = x(z) - \varsigma z^{-1} x(z), \tag{6.16}$$

Applying inverse z-Transform, we get,

$$x_c(k) = x(k) - \varsigma x(k-1). \tag{6.17}$$

The sensor output signal $x(k)$ generated at each sampling instant h is sent to the controller via communication channel. From Eqn. (6.17), it can be noticed that at each sampling instant the effect of fractional delay ($\hat{\tau}_{sc}$) is compensated in the communicated state variable $x_c(k)$ using the immediate past state, the current state and parameter ς. The same communicated state variable $x_c(k)$ is further used to compute the sliding surface.

The mathematical representation of compensated random packet loss within the network is given as,

$$x_p(k) = (1 - \alpha(k))x_c(k) + \alpha(k)x_c(k-1), \tag{6.18}$$

where $x_c(k)$ is the compensated communicated state variable available at the controller and $\alpha(k)\epsilon R$ is the stochastic variable which is represented as Bernoulli's distributed sequence with,

$$Pr\{\alpha(k) = 1\} = E\{\alpha(k)\} = \bar{\alpha}, \tag{6.19}$$

$$Pr\{\alpha(k) = 0\} = 1 - E\{\alpha(k)\} = 1 - \bar{\alpha}, \tag{6.20}$$

where, $0 \preceq \bar{\alpha} \prec 1$ implies the probability that the data packet is lost and $E\{\alpha(k)\}$ is the expectation of the stochastic variable $\alpha(k)$.
Thus, $x_p(k)$ can be written as:

$$x_p(k) = (1 - \bar{\alpha})x_c(k) + \bar{\alpha}x_c(k-1), \tag{6.21}$$

where, $\bar{\alpha}$ is the probability of the data packet lost defined in Eqns. (6.19) and (6.20).
Let the sliding variable that compensate the effect of random fractional delay and packet loss is given by:

$$s(k) = C_s x_p(k), \tag{6.22}$$

where, C_s is the sliding gain which is calculated using discrete LQR method through proper selection of Q and R matrices.
Substituting the value of $x_p(k)$ in Eqn. (6.22) we have:

$$s(k) = (1 - \bar{\alpha})x'_c(k) + \bar{\alpha}x'_c(k-1), \tag{6.23}$$

where,
$x'_c(k) = C_s x(k) - \varsigma C_s x(k-1)$, $x'_c(k-1) = C_s x(k-1) - \varsigma C_s x(k-2)$

This completes the $proof$.

It is assumed that the state packet delay is smaller than sampling interval if the network is free from congestion. So the data packet $x_p(k)$ with delay is used without loss to compute the sliding surface. However, if the network is overloaded due to traffic or congestion, the state packet delay will be larger than sampling interval. At that instance, $x_p(k-1)$ will be used to compute the sliding surface. So from Eqn. (6.23), it can be easily noticed that at each sampling instant the effect of random fractional delay and packet loss in the actual system states are compensated at the sliding surface when it takes the value $x'_c(k-1)$ with probability $\bar{\alpha}$ and $x'_c(k)$ with probability $(1-\bar{\alpha})$.

Once the sliding surface is designed, the next step is to design the discrete-time sliding mode control law which is presented in next section.

6.4 Discrete-Time Networked Sliding Mode Control for NCSs With Random Fractional Delays and Packet Loss

This section presents the derivation of non-switching discrete-time sliding mode control law for NCS using the sliding surface (6.23) as $\mathbf{Theorem-4}$ below.

$\mathbf{Theorem-4}$: The non-switching discrete-time sliding mode controller for system $(6.3, 6.4)$ in the presence of random fractional network delays satisfying (6.5) with packet loss and matched uncertainty $d(k)$ is given as,

$$\begin{aligned} u(k) = -(C_sG)^{-1}(1-\bar{\alpha})^{-1}[Hx(k) - Ix(k) + Kx(k) - Lx(k-1) - \\ J(s(k)) + d_s(k) - d_1] - d(k). \end{aligned} \tag{6.24}$$

where,

$H = (1-\bar{\alpha})(C_sF)$, $I = \varsigma(1-\bar{\alpha})C_s$, $K = \bar{\alpha}C_s$, $L = \varsigma\bar{\alpha}C_s$ and $J = \{1 - q[s(k)]\}$

$\mathbf{Proof}$: Let us define reaching law given by Bartoszewicz and Lesniewski (2014) in the presence of random fractional delay as:

$$s[(k+1)h] = \{1 - q[s(k)]\} - d_s(k) + d_1, \tag{6.25}$$

where $q[s(k)] = \frac{\psi}{\psi+|s(k)|}$ with ψ as user defined constant satisfying $\psi \geq d_2$, d_1 and d_2 are mean and deviation of $d(k)$.

$\mathbf{Remark-6}$: The disturbace $d(k)$ appearing in the reaching law is applied through the network. So, the compensated disturbance $d_s(k)$ using Thiran's approximation is given as:

$$d_s(k) = d(k) - \varsigma d(k-1), \tag{6.26}$$

The reaching law in Eqn. (6.25) indicates that the system states always move towards the specified sliding band given as:

$$|s(k)| \leq \frac{\psi d_2}{\psi - d_2}, \tag{6.27}$$

Substituting the value of $s(k+1)$ in Eqn. (6.25) we get,

$$(1-\bar{\alpha})x'_c(k+1) + \bar{\alpha}x'_c(k) = \{1 - q[s(k)]\} - d_s(k) + d_1,$$

Substituting the value of $x'_c(k+1)$,

$$\begin{aligned}(1-\bar{\alpha})[C_s x(k+1) - \varsigma C_s x(k)] + \bar{\alpha}[C_s x(k) - \varsigma C_s x(k-1)] = \\ \{1 - q[s(k)]\} - d_s(k) + d_1,\end{aligned} \tag{6.28}$$

Further, substituting $x(k+1)$ gives

$$\begin{aligned}(1-\bar{\alpha})[C_s[Fx(k) + G(u(k) + d(k))] - \varsigma C_s x(k)] + \bar{\alpha}[C_s x(k) - \\ \varsigma C_s x(k-1)] = \{1 - q[s(k)]\} - d_s(k) + d_1,\end{aligned} \tag{6.29}$$

Simplifying,

$$\begin{aligned}(1-\bar{\alpha})C_s Fx(k) + (1-\bar{\alpha})C_s G(u(k) + d(k)) - \varsigma(1-\bar{\alpha})C_s x(k) + \\ \bar{\alpha}C_s x(k) - \varsigma\bar{\alpha}C_s x(k-1) = \{1 - q[s(k)]\} - d_s(k) + d_1,\end{aligned} \tag{6.30}$$

Further solving above Eqn. (6.30), the control law can be expressed as:

$$\begin{aligned}u(k) = -(C_s G)^{-1}(1-\bar{\alpha})^{-1}[Hx(k) - Ix(k) + Kx(k) - Lx(k-1) - \\ J(s(k)) + d_s(k) - d_1] - d(k).\end{aligned} \tag{6.31}$$

where,

$H = (1-\bar{\alpha})(C_s F)$, $I = \varsigma(1-\bar{\alpha})C_s$, $K = \bar{\alpha}C_s$, $L = \varsigma\bar{\alpha}C_s$ and $J = \{1 - q[s(k)]\}$.

This completes the **proof**.

Similarly, the effect of random fractional delay at controller to actuator and packet loss can be compensated at the actuator end. The compensated control signal applied to the plant is given by:

$$u_a(k) = (1-\bar{\beta})(u_c(k)) + \bar{\beta}(u_c(k-1)), \tag{6.32}$$

where,
$u_c(k) = u(k) - \gamma' u(k-1)$ and $u_c(k-1) = u(k-1) - \gamma' u(k-2)$ and $\gamma' = \frac{\hat{\tau}_{ca}}{1+\hat{\tau}_{ca}}$ and $\hat{\tau}_{ca}$ is the random fractional delay from controller to actuator.
From Eqn. (6.32) it can be easily inferred that at each sampling instant the effect of random fractional delay from controller to actuator is compensated in $u_c(k)$ using past control signal, present control signal and parameter 'γ'' while packet loss is compensated which takes the value $u_c(k-1)$ with probability $\bar{\beta}$ and $u_c(k)$ with probability $(1-\bar{\beta})$.

The next section discusses about the stability condition for the closed loop system such that the system states remain within specified band (6.27) using control law (6.31).

6.4.1 Stability Analysis

The trajectories of the closed loop system given in Eqn. (6.3,6.4) drive towards the sliding surface as mentioned in Eqn. (6.23) for a given controller in Eqn. (6.31) in the presence of total networked delay $\hat{\tau}$ satisfying Eqn. (6.5), packet loss condition satisfying $0 \preceq \bar{\alpha} \prec 1$ and matched uncertainty satisfying (3.7) such that the following condition in Eqn. (6.33) must exists:

$$\rho s^T(k)s(k) \succ 0. \tag{6.33}$$

Proof: Consider the compensated sliding surface (6.23),

$$s(k) = (1-\bar{\alpha})x'_c(k) + \bar{\alpha}x'_c(k-1). \tag{6.34}$$

Let us defined Lyapunov function as,

$$V_s(k) = s^T(k)s(k). \tag{6.35}$$

Taking the forward difference we have,

$$\Delta V_s(k) = s^T(k+1)s(k+1) - s^T(k)s(k). \tag{6.36}$$

Using Eqn. (6.34) we get,

$$\Delta V_s(k) = [(1-\bar{\alpha})x'_c(k+1)+\bar{\alpha}x'_c(k)]^T[(1-\bar{\alpha})x'_c(k+1)+\bar{\alpha}x'_c(k)]-s^T(k)s(k). \tag{6.37}$$

Substituting the value of $x'_c(k+1)$ we get,

$$\Delta V_s(k) = [(1-\bar{\alpha})[C_s x(k+1) - \alpha' C_s x(k)] + \bar{\alpha}[C_s x(k) - \alpha' C_s x(k-1)]]^T \tag{6.38}$$
$$[(1-\bar{\alpha})[C_s x(k+1) - \alpha' C_s x(k)] + \bar{\alpha}[C_s x(k) - \alpha' C_s x(k-1)]] - s^T(k)s(k).$$

Further substituting the value of $x(k+1)$,

$$\Delta V_s(k) = [(1-\bar{\alpha})[C_s[Fx(k) + G(u(k)+d(k))] - \alpha' C_s x(k)] + \bar{\alpha}[C_s x(k) - \tag{6.39}$$
$$\alpha' C_s x(k-1)]]^T[(1-\bar{\alpha})[C_s[Fx(k) + G(u(k)+d(k))] - \alpha' C_s x(k)] +$$
$$\bar{\alpha}[C_s x(k) - \alpha' C_s x(k-1)]] - s^T(k)s(k).$$

Substituting $u(k)$ and rewritting above Eqn. (6.39),

$$\Delta V_s(k) = \Gamma - s^T(k)s(k). \tag{6.40}$$

where, $\Gamma = [(1-\bar{\alpha})^{-1}[1-q(s(k))]s(k) - d_s(k) + d_1]^T[(1-\bar{\alpha})^{-1}[1-q(s(k))]s(k) - d_s(k) + d_1]$. The term Γ can be tuned closed to zero by appropriately selecting the parameter ψ and $\bar{\alpha}$. If Γ is close to zero then $s^T(k)s(k)$ will be larger than Γ. Thus for any small parameter ρ we have,

$$\Gamma - s^T(k)s(k) \prec \rho s^T(k)s(k). \tag{6.41}$$

Tuning of parameter Γ, leads to, $\Delta V_s(k) \prec \rho s^T(k)s(k)$ which guarantees the convergence of $\Delta V_s(k)$ and implies that any trajectory of the system (6.3,6.4) will be driven onto the sliding surface and maintain on it.

This completes the **proof**.

6.5 Results and Discussions

In this section the effect of proposed control algorithm is validated through simulation and experimental results carried out in the presence of random fractional delay, packet loss and matched uncertainty. An illustrative example given by Wu and Chen (2007) is considered for simulation while Quanser DC motor is used for experimental purpose.

6.5.1 Simulation Results

Consider the continuous-time LTI system as mentioned in (4.4.1),

$$\dot{x}(t) = Ax(t) + Bu(t-\tau_r) + Dd(t), \tag{6.42}$$

$$y(t) = Cx(t), \tag{6.43}$$

where,
$A = \begin{bmatrix} -0.7 & 2 \\ 0 & -1.5 \end{bmatrix}$, $B = \begin{bmatrix} -0.03 \\ -1 \end{bmatrix}$,
$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Dsicretizing the above system with sampling interval of $h = 30msec$ we get,

$$x(k+1) = Fx(k) + Gu(k - \hat{\tau}) + d(k), \tag{6.44}$$

$$y(k) = Cx(k), \tag{6.45}$$

where,
$F = \begin{bmatrix} 0.9792 & 0.05805 \\ 0 & 0.956 \end{bmatrix}$, $G = \begin{bmatrix} -0.001771 \\ -0.02934 \end{bmatrix}$,
$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

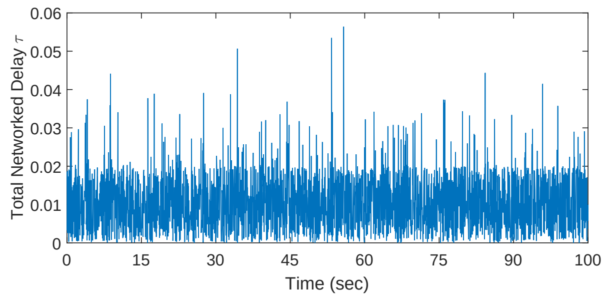


Figure 6.2: Total networked fractional delay τ

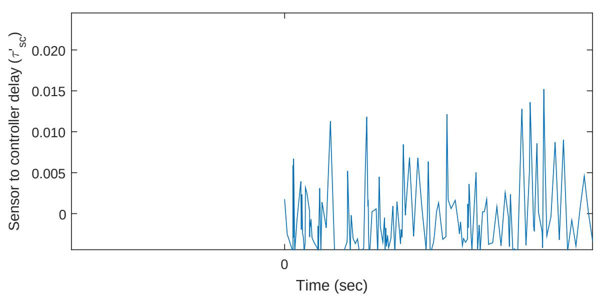


Figure 6.3: Magnified plot for sensor to controller fractional delay

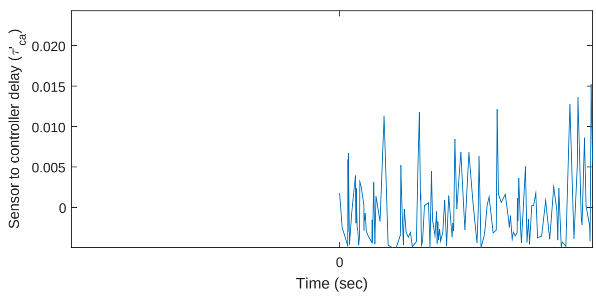


Figure 6.4: Magnified plot for controller to actuator fractional delay

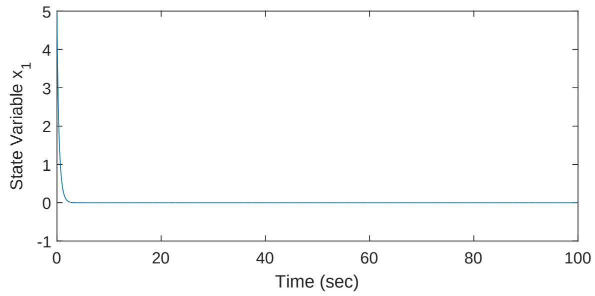


Figure 6.5: State variable $x_1(k)$

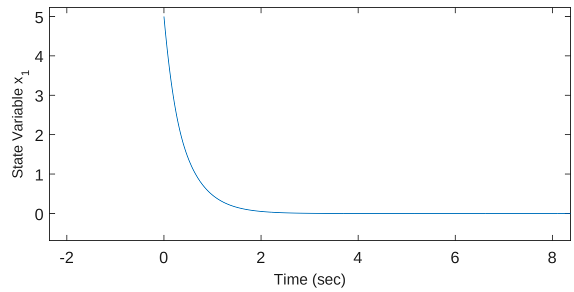


Figure 6.6: Magnified state variable $x_1(k)$

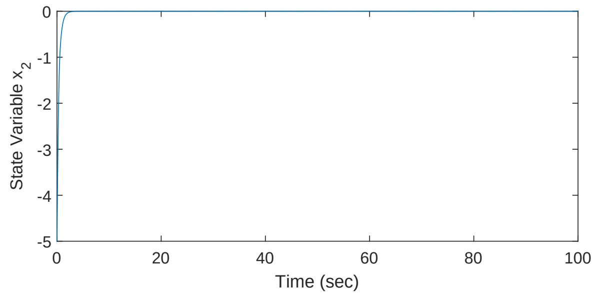


Figure 6.7: State variable $x_2(k)$

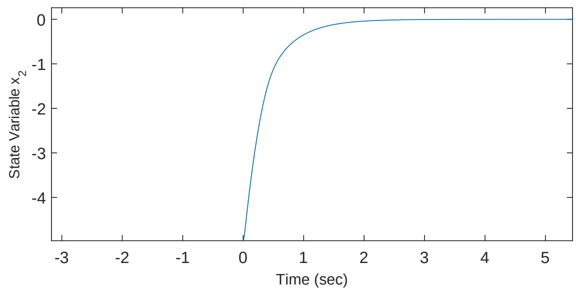


Figure 6.8: Magnified state variable $x_2(k)$

Figures (6.2) to (6.22) show the nature of the system under networked environment with random fractional delays, packet loss situation and matched uncertainty. In order to prove the robustness of the proposed algorithm in the presence of variable networked delay and packet loss the slow time varying disturbance is applied to the system as shown in Figure (4.1). The random nature of total networked induced fractional

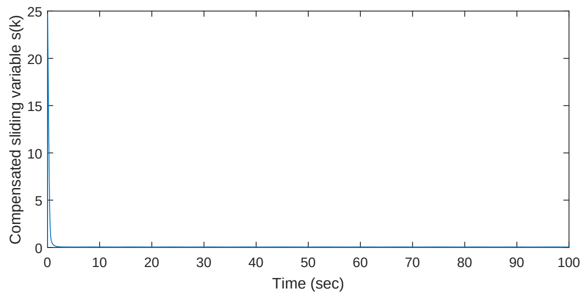


Figure 6.9: Compensated sliding variable $s(k)$

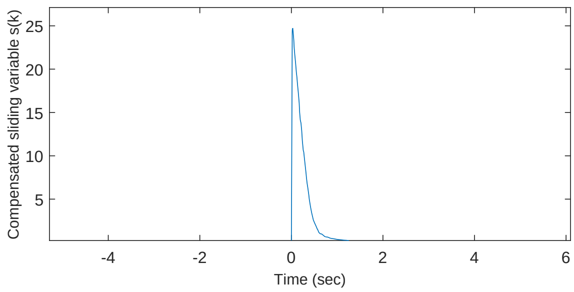


Figure 6.10: Magnified compensated sliding variable $s(k)$

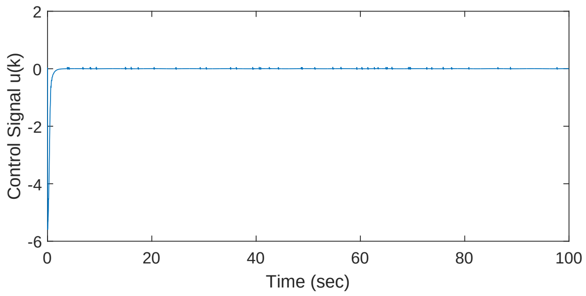


Figure 6.11: Control Signal $u(k)$

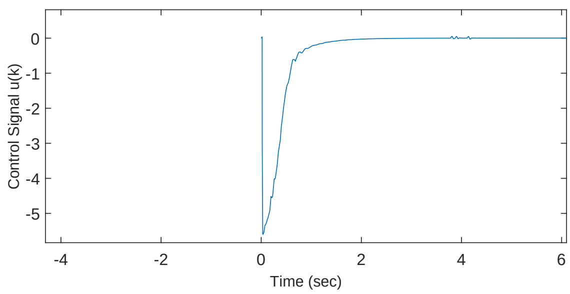


Figure 6.12: Magnified control signal $u(k)$

delay is modelled using Poisson's distribution. It is assumed that at every sampling instant one event is generated considering total networked induced delay lesser than sampling interval with zero trial. So, according to Poisson's distribution under these assumptions the probability of networked delays lesser than sampling interval will be $p = 0.63$ while the probability of the networked delays greater than sampling interval

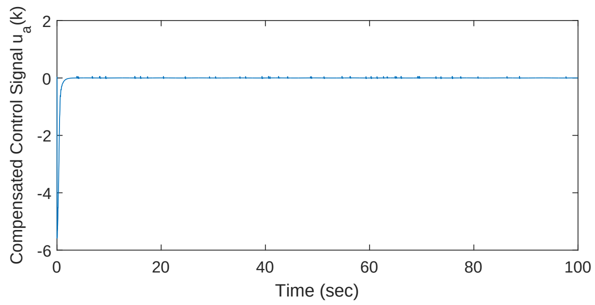


Figure 6.13: Compensated control signal $u_a(k)$

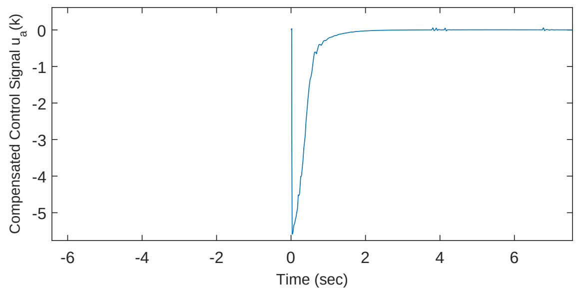


Figure 6.14: Magnified compensated control signal $u_a(k)$

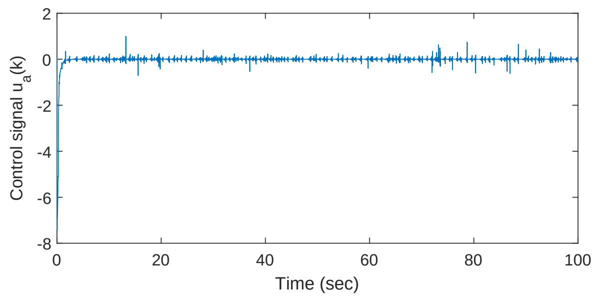


Figure 6.15: Compensated control signal $u_a(k)$ with 10% packet loss

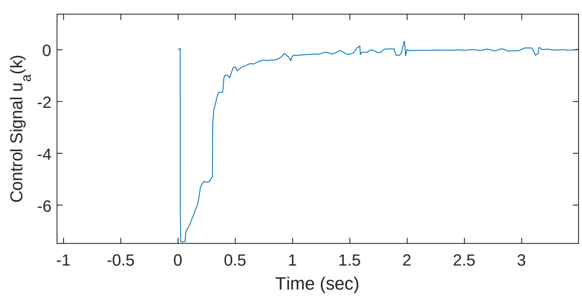


Figure 6.16: Magnified compensated control signal $u_a(k)$ with 10% packet loss

is $1 - p = 0.37$. Thus, the total network induced fractional delay generated within the network is $0.003sec \leq \tau' \leq 0.055sec$ respectively. Figure (6.2) depict the random nature of total networked induced fractional delays modelled using Poisson's distribution. Figures (6.3) and (6.4) show the magnified results of sensor to controller fractional delay and controller to actuator fractional delay. As discussed in previous chapter, the

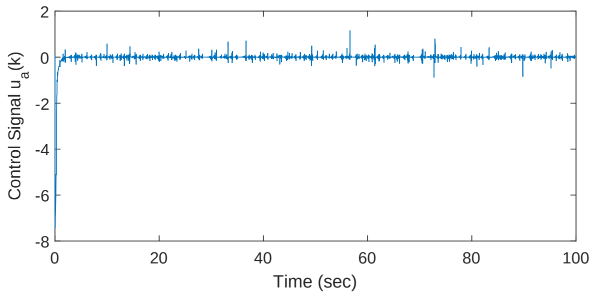


Figure 6.17: Compensated control signal $u_a(k)$ with 30% packet loss

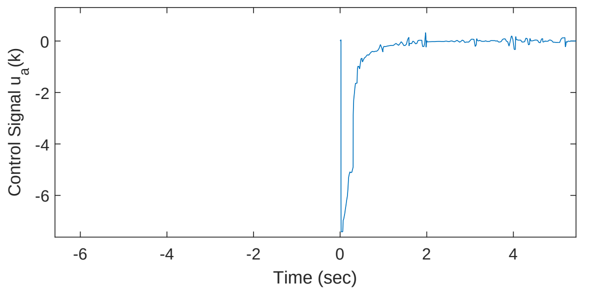


Figure 6.18: Magnified compensated control signal $u_a(k)$ with 30% packet loss

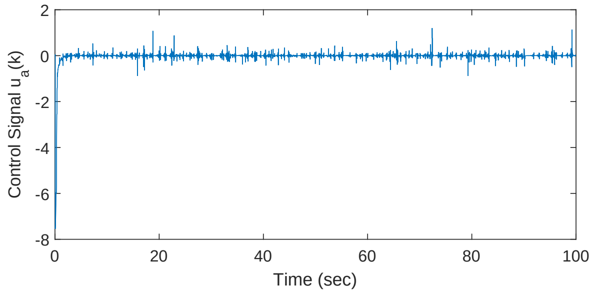


Figure 6.19: Compensated control signal $u_a(k)$ with 50% packet loss

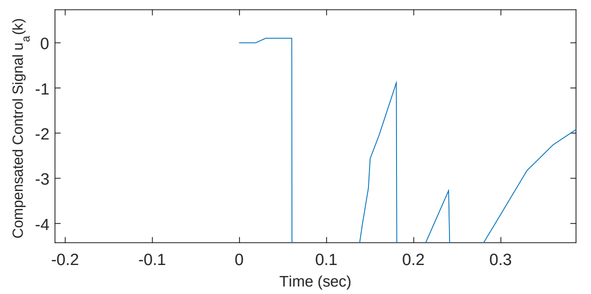


Figure 6.20: Magnified compensated control signal $u_a(k)$ with 50% packet loss

processing delays and computational delays ocuuring at sensor, actuator and controller are neglected due to its negligible effect.

The sliding gain C_s is calculated using discrete LQR method with $Q = diag(1500, 1000)$ and $R = 1$. The computed values of sliding gain comes out to be $C_s = [-1.577 \quad -2.456]$. The Quasi-sliding mode band comes out to be $|s(k)| \preceq +0.1$ to -0.1 with proper se-

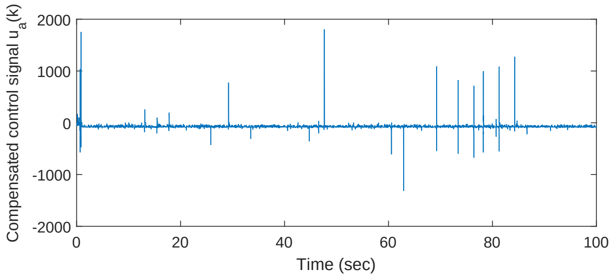


Figure 6.21: Compensated control signal $u_a(k)$ with fractional delay greater than sampling interval

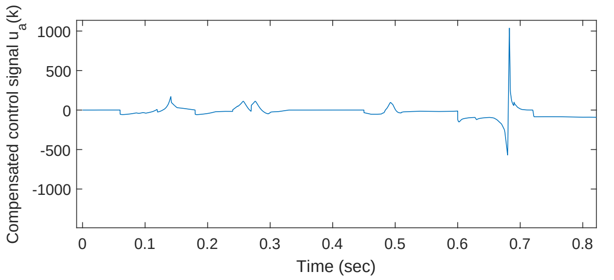


Figure 6.22: Magnified compensated control signal $u_a(k)$ with fractional delay greater than sampling interval

lection of user defined constant $\psi = 10$.

The simulation results are discussed in three parts: (i) Figures (6.5) to (6.14) discuss the effect of proposed control algorithm for specified networked delay range under single packet transmission (ii) Figures (6.15) to (6.20) describes the effect of fractional delays on compensated control signal for different packet loss situations and (iii) Figures (6.21) and (6.22) show the results of compensated control signal computed at actuator side when fractional delays are greater than the sampling interval.

Figures (6.5) and (6.7) show the results of system state variables with initial condition $x(k) = [5 \quad -5]$. It can be noticed that both the state variables slide towards the origin from given initial condition in the presence of random fractional delays. In order to show the accurate effect of random fractional delay compensation at the output, results are magnified as shown in Figures (6.6) and (6.8) respectively. It can be observed that in both the cases the state variables are computed from initial sampling. Thus, the effect of random fractional delay from sensor to controller and controller to actuator is compensated using the proposed algorithm. The same effect of compensation is observed in compensated sliding variable (Figure (6.9)), control signal (Figure (6.11)) and compensated control signal (Figure (6.13)) respectively. On observation of the magnified results in Figures (6.10), (6.12) and (6.14) it can be noticed that all the three parameters are computed from first sampling instant even in the presence of sensor to controller delay and controller to actuator delay. The amount of delays generated at both sides of network at that sampling instant are indicated in Figures (6.3) and (6.4) respectively. Thus the effects of random fractional delay from sensor to controller are compensated in the sliding surface and remains within the specified band (6.27) while controller to actuator random fractional delay is compensated at the actuator end. The efficacy of the proposed algorithm was further tested under packet loss situation for different probabilities. Figures (6.15), (6.17) and (6.19) show the results of compensated control signal for different values of $\bar{\alpha} = 0.10, 0.30$ and 0.50 respectively. It can be noticed that when the packet loss probability within the network is 50% the system shows unacceptable response with high frequency oscillations. Thus it can be noticed that when the packet loss is generated within the network the robust terms will generate more action to stabilize the system which in turn makes the compensated control signal oscillatory in nature. Figures (6.16), (6.18) and (6.20) show the magnified results of the compensated control signal for a given set of packet loss probabilities. It can be observed that when the packet loss probability increases to 50% within the network the proposed algorithm cannot compensate the effect of fractional delay as compared to the cases of 10% and 30% packet losses. Figures (6.21) and (6.22) show the nature of compensated control signal when the probabilities of the random fractional delays are greater than the sampling interval. It can be observed that when the probabilities of network fractional delays are reversed than previous case that is $p = 0.37$ and $1 - p = 0.63$ the proposed technique cannot compensate the effect of random fractional delays and the system response becomes unstable with high frequency oscillations generated at the output. Thus, from above results it can be concluded that the proposed technique works efficiently with random networked delay of $0.003sec \leq \hat{\tau} \leq 0.055sec$ in simulated environment. The proposed controller compensates the effect of random networked delay for $\psi = 10$ and $\bar{\alpha} = 0.3$ satisfying Eqn. (6.5) and shows the stable response satisfying condition mention in Eqn. (6.33) in the presence of random fractional delays, packet loss and matched uncertainty.

6.5.2 Experimental Results

The state space form of DC Motor plant mentioned in (3.6.1) using Eqn. (3.34) is given as,

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_r) + Dd(t), \tag{6.46}$$
$$y(t) = Cx(t), \tag{6.47}$$

where,
$A = \begin{bmatrix} -201 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, , $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Dsicretizing the above system with sampling interval of $h = 30msec$ we get,

$$x(k+1) = Fx(k) + Gu(k - \hat{\tau}) + d(k), \tag{6.48}$$
$$y(k) = Cx(k), \tag{6.49}$$

where,
$F = \begin{bmatrix} 0.001836 & 0 \\ 0.004573 & 1 \end{bmatrix}$, $G = \begin{bmatrix} -0.004753 \\ -0.0001242 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

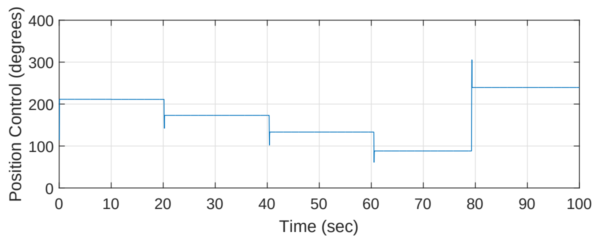


Figure 6.23: Results of position control of DC Motor with 10% packet loss

In this section, experimental results are briefly discussed with DC motor as a plant in the presence of random fractional delays, packet loss and matched uncertainty situations. The position of DC motor is considered as a reference signal. The variable

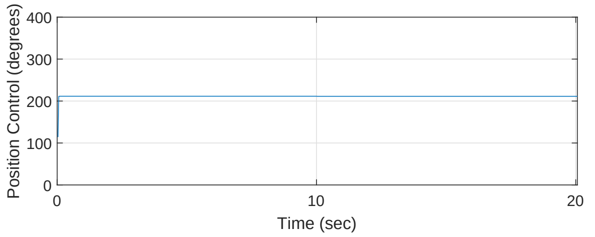


Figure 6.24: Result of magnified position control of DC Motor with 10% packet loss

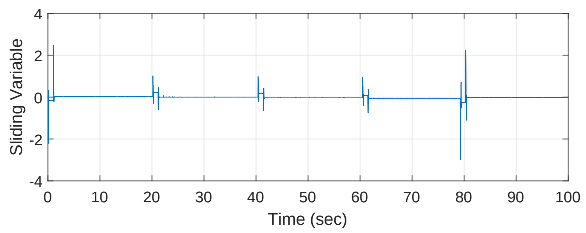


Figure 6.25: Result of compensated sliding variable with 10% packet loss

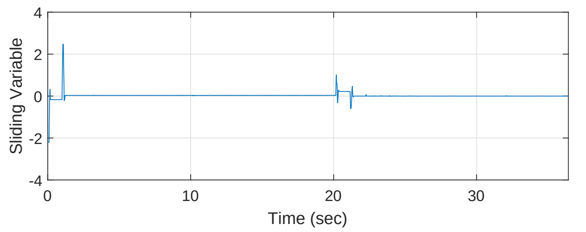


Figure 6.26: Magnified compensated sliding variable with 10% packet loss

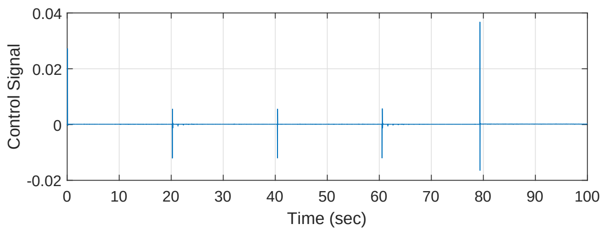


Figure 6.27: Control signal with 10% packet loss

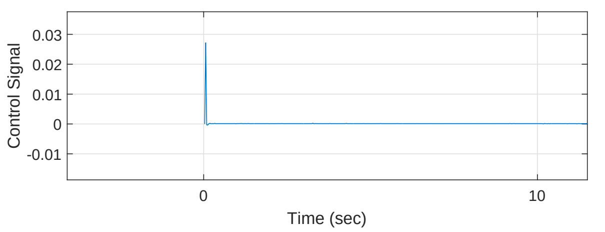


Figure 6.28: Magnified control signal with 10% packet loss

fractional delays are computed using Poisson's distribution considering the same assumptions as mentioned in the simulation results section. The values of sliding gain parameter C_s, sliding band $|s(k)|$ and user defined constant ψ are same in order to study the effect of proposed control algorithm on the real time system. In simulation section

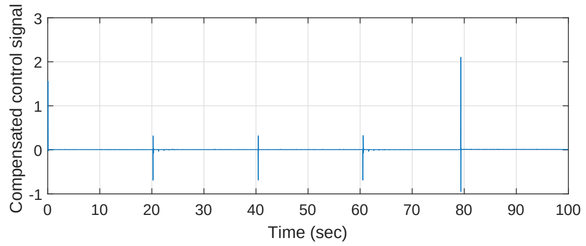


Figure 6.29: Compensated control signal $u_a(k)$ with 10% packet loss

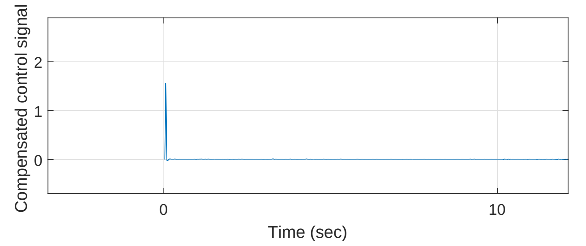


Figure 6.30: Magnified compensated control signal $u_a(k)$ with 10% packet loss

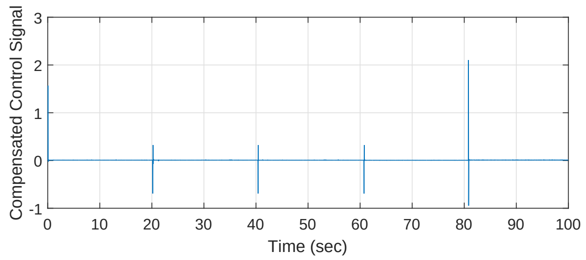


Figure 6.31: Compensated control signal $u_a(k)$ with 30% packet loss

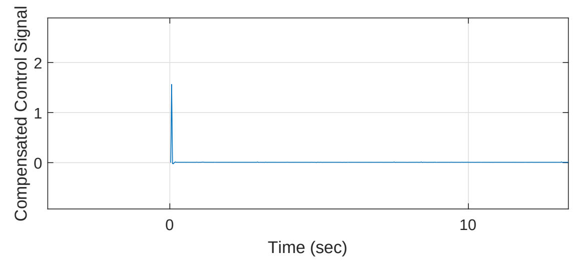


Figure 6.32: Magnified compensated control signal $u_a(k)$ with 30% packet loss

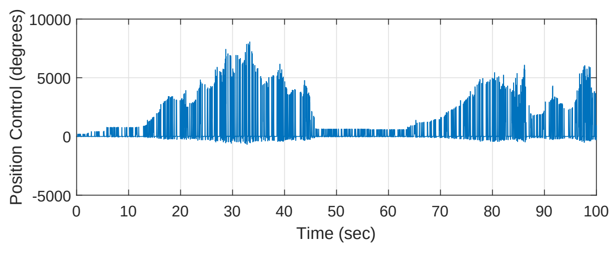


Figure 6.33: Tracking Response with network delays greater than sampling interval

the effects of control algorithm are well explained under two different conditions (i) in the absence of packet loss and (ii) in the presence of packet loss. But, when any system is connected to real time networks there are very few chances of secure communication

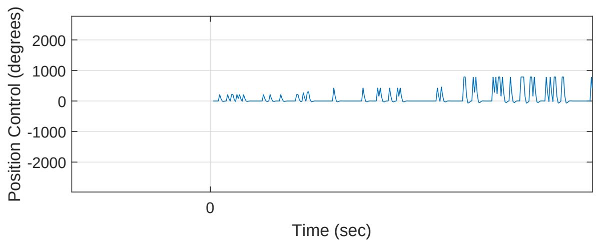


Figure 6.34: Magnified tracking response with network delays greater than sampling interval

between the plant and controller in the terms of data transfer. Some of the data packets will be lost due to various reasons such as jitter, congestion or traffic problems within the network. So, in real time application it is essential to study the effect of control algorithm in the presence of packet loss and random time delay. Figures (6.23) to (6.34) shows the nature of the DC motor plant in terms of reference tracking, compensated sliding variable, control signal and compensated control signal for specified random fractional delays shown in Figure (6.2), matched uncertainty shown in Figure (4.1) with probability of single packet loss $\bar{\alpha} = 0.1$. Figure (6.23) shows the tracking response of the DC motor. It can be observed that the position of DC motor is controlled accurately in the presence of random fractional delays and packet loss situation. In order to show the effect of time delay compensation the magnified tracking result is shown in Figure (6.24). It can be noticed that the effect of total network fractional delay is compensated as the output tracks the reference signal at first sampling instant. The same effect of fractional delay compensation is observed in compensated sliding variable and control signal as shown in Figures (6.25) and (6.27) respectively. Observing the magnified results shown in Figures (6.26) and (6.28), it can be noticed that both the parameters are computed from first sampling instant. Thus the effect of random fractional delay from sensor to controller is compensated at the sliding surface. The magnified result of random fractional delay from sensor to controller is shown in Figure (6.3). Figure (6.29) shows the nature of compensated control signal. It can be noticed that the effect of random fractional delay from controller to actuator is compensated at the actuator side and converges to zero rapidly without increasing the amplitude. The magnified result of the same in Figure (6.30). The magnified result of controller to actuator delay is shown in Figure (6.4). The efficacy of the proposed algorithm was further tested by increasing the packet loss probability by three times that is, $\bar{\alpha} = 0.3$ for specified network delays and matched uncertainty. Observing the results of compensated control signal as shown in

Figures (6.31) and (6.32), it can be noticed that the effect of fractional delay at the actuator side is still compensated with packet loss probability of 30%. Figure (6.33) shows the nature of tracking response when random fractional delay is greater than sampling interval. It can be noticed that the performance of the system goes to unstable condition and does not compensate the effect of random fractional delays. The magnified result of the same is shown in Figure (6.34).

Thus from above implementation results, it can be noticed that the proposed control algorithm proves to be robust and efficient controller as it shows the stable response satisfying condition (6.33) and compensates the effect of specified random fractional delays satisfying (6.5) in presence of packet loss and matched uncertainty.

6.6 Conclusion

In this chapter, we proposed the discrete-time SMC (Non-Switching) algorithm in the presence of random fractional delay and packet loss in the presence of uncertainty. The random fractional delay is modelled using Poisson's distribution and packet loss is modelled using Bernoulli's distribution function. The random fractional delay in forward and feedback channels are compensated by Thiran Approximation in the sliding surface and actuator end respectively. A novel sliding surface is designed using Thiran's Approximation. A non-switching type discrete-time sliding mode controller is designed such that system states slide along the proposed compensated surface and maintain within the specified band. The stability condition of closed loop NCSs is derived using Lyapunov approach that ensures finite time convergence of system states in presence of network non-idealities. The effectiveness of the proposed algorithm is examined under different possible conditions through illustrative example as well as real time plant. The results proved that the control law derived using Thiran's Approximation compensates the random fractional delay precisely even in the presence of 30% packet loss as well as networked delay having values greater than sampling interval.

CHAPTER 7

DISCRETE-TIME NETWORKED SLIDING MODE CONTROL (DNSMC) WITH MULPTIPLE PACKET TRANSMISSION POLICY

7.1 Introduction

In communication domain, generally the data transfer between two devices takes place in the form of frames or packets. It mainly depends upon the distance of communication. When the distances are shorter the data are transmitted in the form of frames. However, when the distances are longer the same frame is breakdown in the form of small packets in order to have secure and reliable communication. In NCSs the data transfer takes place over the longer distance. So, those data in the form of frames are converted into small packets which are defined as multiple packets. When such packets are lost during transmission it is defined as multiple packet loss. It is necessary to study the effect of such lost packets during transmission. In this chapter, a mathematical model is derived for multiple packet loss using probability function approach. The random time delay and multiple packet loss are treated separately in order to study the outcome of both parameters on the system. The random communication delay and multiple packet loss in the forward and feedback channel are compensated through Thiran approximation and probability distribution respectively. Based on the proposed approach the sliding surface is designed and discrete-time sliding mode control law is derived that computes the control actions in the presence of random network delay and multiple packet loss. The stability of the closed loop NCS is also derived using Lyapunov approach that assures the finite time convergence in the presence of network non-idealities. The efficacy of the proposed algorithm is examined through simulation and experimental results in the presence of random communication fractional delays and multiple packet loss.

7.2 Problem Formulation

Figure (7.1) depicts the block diagram of networked control system with multiple packets transmission. It can be observed that the state information and control information available through sensor and controller are splitted in the form of small packets and transmitted through the network. These transmitted packets will suffer from sensor to controller random delay and controller to actuator random delay. During multiple packets transmission it is necessary to consider three different situations (i) none of the packets are lost (ii) any one or more than one packet is lost or (iii) all the packets are lost. If second or third situation arises during transmission the frame structure would be incomplete at the controller side and the false control actions will be generated which affects the performance of the system. The multiple packets received at the controller side and actuator side are converted in the form of frames having fixed length and after processing in the form of small packets they are transmitted back to the networks.

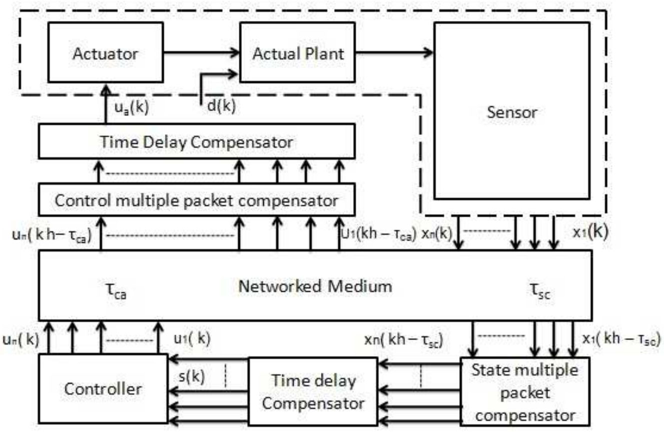


Figure 7.1: Block diagram of NCS with multiple packets transmission

In order to study the effect of multiple packet loss under random time delay compensation on the system it is necessary to developed the mathematical model of mutiple packet loss in the forward and feedback channel. The next segment describes the mathematical model of mutiple packet loss that is generated from sensor to controller and controller to actuator. The concept of probability distribution function is used to modelled multipe packet loss.

Multiple Packet Loss Model From Sensor to Controller and Controller to Actuator

As discussed earlier, in mutiple packet loss model the frames are splitted in the form of small packets. Let, the plants states are splitted into r equal parts given as:

$$x(k) = \left[x_1^T(k) \quad x_2^T(k) \quad ... \quad x_r^T(k), \right]^T \tag{7.1}$$

and every part of plant state is lumped into packet.
Similarly, let the control signal is splitted into s equal parts given as:

$$u(k) = \left[u_1^T(k) \quad u_2^T(k) \quad ... \quad u_s^T(k), \right]^T \tag{7.2}$$

and every part of control state is lumped into packet. where $x_i(k)\epsilon R^{r_i}$, $r_i\epsilon Z^+$ and $\Sigma_{i=1}^r = n$ and $u_i(k)\epsilon R^{s_i}$, $s_i\epsilon Z^+$ and $\Sigma_{i=1}^s = p$.
For, simplicity we consider the case that the plant states and control signals are splitted

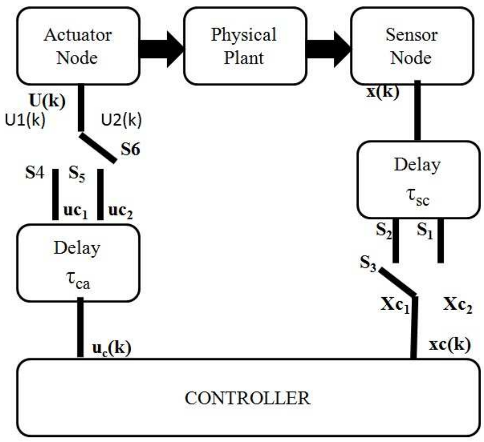


Figure 7.2: Schematic Diagram of NCSs with state data frame splitted in two parts

in two parts. Figure (7.2) describes the schematic diagram of networked control system with multiple packet transmission with state data frames splitted in two equal parts. As shown in Figure (7.2) two different cases are consider on each side of the network medium in form of switch position S_1 to S_6. When the switch position is at S_3 or S_6 none of the packets will be lost or either all the packet will be lost at the controller and actuator side but when the switch is at position S_1 or S_2 and S_4 or S_5 anyone packet is lost on either side of the channel. As shown in Figure (7.2) at sampling instant h, the state data packets available at the input of the controller will be $Xc_1(k)$ and $Xc_2(k)$ respectively.

Thus the process of state data transmission can be described as follows:

$\mathbf{Case - I}$: If the switch is in position S_3 then,

$$Xc_1(k) = \begin{cases} x_1(k-1); & \text{if } r_1 \prec P_{loss1} \\ x_1(k); & \text{if otherwise} \end{cases}$$

$$Xc_2(k) = \begin{cases} x_2(k-1); & \text{if } r_2 \prec P_{loss2} \\ x_2(k); & \text{if otherwise} \end{cases}$$

$\mathbf{Case - II}$: If the switch is in position S_2 then,

$$Xc_1(k) = x_1(k)$$

$$Xc_2(k) = \begin{cases} x_2(k-1); & \text{if } r_2 \prec P_{loss2} \\ x_2(k); & \text{if otherwise} \end{cases}$$

$\mathbf{Case - III}$: If the switch is in position S_1 then,

$$Xc_1(k) = \begin{cases} x_1(k-1); & \text{if } r_1 \prec P_{loss1} \\ x_1(k); & \text{if otherwise} \end{cases}$$

$$Xc_2(k) = x_2(k)$$

where, $0 \prec P_{loss1} \prec 1$ and $0 \prec P_{loss2} \prec 1$ is the probability of multiple state packet loss over the network and r_1, r_2 are the random variables uniformly distributed over the interval [0,1].

It is also assumed that the measured outputs, which are equivalent to states of the system are sent to the controller via communication channel. As a result, the measurement may involve randomly varying communication delays. The communicated state variable over the network having the random delays from sensor to controller is given by:

$$x_c(k) = \begin{bmatrix} Xc_1(k-\hat{\tau}_{sc}) \\ Xc_2(k-\hat{\tau}_{sc}) \end{bmatrix},$$

simplyfying it we get,

$$x_c(k) = Z_1 Xc_1(k-\hat{\tau}_{sc}) + Z_2 Xc_2(k-\hat{\tau}_{sc}), \tag{7.3}$$

where,

$Z_1 = diag(I_{r_1}, 0)$ and $Z_2 = diag(I_{r_2}, 0)$.

Thus the generalized equation is given by:

$$x_c(k) = \Sigma_{i=0}^{r} Z_i x_i(k - \hat{\tau}_{sc}), \tag{7.4}$$

where, $Z_i = diag(0, ..., I_{r_i}, ...0)$, $x_c(k)$ is the communicated state variable over the network and $\hat{\tau}_{sc}$ is sensor to controller random fractional delay.

Similarly, the process of control data transmission can be described as follows:

$\mathbf{Case - IV}$: If the switch is in position S_6 then,

$$U_1(k) = \begin{cases} u_1(k-1); & \text{if } r_3 \prec P_{loss3} \\ u_1(k); & \text{if otherwise} \end{cases}$$

$$U_2(k) = \begin{cases} u_2(k-1); & \text{if } r_4 \prec P_{loss4} \\ u_2(k); & \text{if otherwise} \end{cases}$$

$\mathbf{Case - V}$: If the switch is in position S_5 then,

$U_1(k) = u_1(k)$

$$U_2(k) = \begin{cases} u_2(k-1); & \text{if } r_4 \prec P_{loss4} \\ u_2(k); & \text{if otherwise} \end{cases}$$

$\mathbf{Case - VI}$: If the switch is in position S_4 then,

$$U_1(k) = \begin{cases} u_1(k-1); & \text{if } r_3 \prec P_{loss3} \\ u_1(k); & \text{if otherwise} \end{cases}$$

$U_2(k) = u_2(k)$

where, $0 \prec P_{loss3} \prec 1$ and $0 \prec P_{loss4} \prec 1$ is the probability of the multiple control packet loss over the network and r_3, r_4 are the random variable uniformly distributed over the interval [0,1].

The measured outputs, which are equivalent to control signal of the system are sent to the actuator via communication channel. As a result, the measurement may involve randomly varying communication delay. The communicated controlled variable over the network having the random delays from controller to actuator is given by:

$$u_a(k) = \begin{bmatrix} U_1(k - \hat{\tau}_{ca}) \\ U_2(k - \hat{\tau}_{ca}) \end{bmatrix},$$

Simplyfying it we get,

$$u_a(k) = D_1 U_1(k - \hat{\tau}_{ca}) + D_2 U_2(k - \hat{\tau}_{ca}), \tag{7.5}$$

where,
$D_1 = diag(I_{s_1}, 0)$ and $D_2 = diag(I_{s_2}, 0)$.
Thus the generalized equation is given by:

$$u_a(k) = \Sigma_{j=0}^{s} D_j u_j(k - \hat{\tau}_{ca}), \tag{7.6}$$

where, $D_j = diag(0, ..., I_{s_j}, ...0)$, $u_a(k)$ is the control signal available at actuator side and $\hat{\tau}_{ca}$ is controller to actuator random fractional delay.
$\mathbf{Remark - 9}$: In this chapter nature of the system, total random fractional network delays $\hat{\tau}$, sensor to controller random fractional delay $\hat{\tau}_{sc}$ and controller to actuator random fractional delay $\hat{\tau}_{ca}$ would remain same as described in section (6.2).

$\mathbf{Problem\ Statement}$: The main objective, is to design the discrete-time sliding mode control law for system (6.3,6.4) in the presence of random fractional delay $\hat{\tau}_{sc}$ and $\hat{\tau}_{ca}$ with multiple packet loss situation and matched uncertainty satisfying (6.5).

Once the mathematical model of multiple packet loss is derived the next step is to design the compensated sliding surface and discrete-time control law that compensates the effect of random fractional delays in the presence of multiple packet loss and matched uncertainty.

7.3 Design of Sliding Surface With Multiple Packet Loss

This section describes the designing of compensated sliding surface under multiple packet loss presented in the form of $\mathbf{Lemma - 4}$.
$\mathbf{Lemma - 4}$: The compensated sliding surface for the given system (6.3, 6.4) with sensor to controller random fractional delay under multiple packet loss and matched

uncertainty satisfying condition (6.5) and (3.7) is given as:

$$s(k) = \Sigma_{i=0}^{r}[C_s Z_i x_i(k) - \varsigma C_s Z_i x_i(k-1)], \tag{7.7}$$

where, C_s is the sliding gain, ς is the parameter designed using Thiran Approximation.
Proof: The sliding variable with sensor to controller random fractional delay and mutiple packet loss is given as:

$$s(k) = C_s x_c(k). \tag{7.8}$$

Substituting $x_c(k)$ from Eqn. (7.4) to Eqn. (7.8) we get,

$$s(k) = \Sigma_{i=0}^{r} C_s Z_i x_i(k - \hat{\tau}_{sc}), \tag{7.9}$$

where, $\hat{\tau}_{sc}$ is random fractional delay from sensor to controller.
The random fractional delay from sensor to controller $(\hat{\tau}_{sc})$ is model using Poisson's distribution function defined in Eqns. (6.9) and (6.10) respectively.
Applying z-transform to above Eqn. (7.9) we get,

$$s(k) = \Sigma_{i=0}^{r} C_s Z_i [x_i(z) z^{-\hat{\tau}_{sc}}]. \tag{7.10}$$

Using Thiran Approximation and Eqn. (6.14), $z^{-\hat{\tau}_{sc}}$ is given as

$$s(k) = \Sigma_{i=0}^{r} C_s Z_i x_i(z)[1 - \varsigma z^{-1}], \tag{7.11}$$

where, $\varsigma = \frac{\hat{\tau}_{sc}}{\hat{\tau}_{sc}+1}$.
Applying inverse z-transform to above Eqn. (7.11) we get,

$$s(k) = \Sigma_{i=0}^{r} C_s Z_i [x_i(k) - \varsigma x_i(k-1)]. \tag{7.12}$$

Further simplification,

$$s(k) = \Sigma_{i=0}^{r}[C_s Z_i x_i(k) - \varsigma C_s Z_i x_i(k-1)]. \tag{7.13}$$

This completes the $proof$.

The above Eqn. (7.13) represents compensated sliding surface with multiple packet loss. It is noticed that at each sampling interval the effect of delayed sensor packets are compensated through immediate past state data packets, current state data packets and parameter ς. While the effect of multiple packet loss is compensated through probability distribution function. If random variable r is greater than packet loss probabilities then the data packet $x_i(k)$ is received and, if random variable r is lesser than packet loss probabilities then $x_i(k-1)$ will be received at the controller.

The next section represents the design of discrete-time sliding mode control law along with stability analysis in the presence of multiple packet loss and matched uncertainty.

7.4 Discrete-Time Sliding Mode Control Law With Random Fractional Delay and Multiple Packet Loss

This section discusses the derivation of discrete-time sliding mode control law for NCS using sliding surface (7.13) in the form of $\mathbf{Theorem-5}$.

$\mathbf{Theorem-5}$: The discrete-time sliding mode control law for system (6.3,6.4) in the presence of sensor to controller random fractional delay $(\hat{\tau}_{sc})$, multiple packet loss and matched uncertainty $d(k)$ satisfying (3.7) is given as,

$$u(k) = -(C_s G')^{-1}[Hx_i(k) - Ix_i(k) - J(s(k)) + d_s(k) - d_1] - d(k). \tag{7.14}$$

where,

$G' = \Sigma_{i=0}^{r}(Z_i G)$, $H = \Sigma_{i=0}^{r}(C_s Z_i F)$, $I = \Sigma_{i=0}^{r}\alpha' C_s Z_i$, and $J = \{1 - q[s(k)]\}$.

$\mathbf{Proof}$: Let us define non-switching reaching law described in (6.25) considering the same parameters in the presence of network fractional delay as:

$$s[(k+1)h] = \{1 - q[s(k)]\} - d_s(k) + d_1. \tag{7.15}$$

The above all parameters such as $\{q[s(k)]\}, d_s(k), d_1$, condition of user defined constant ψ and sliding band $|s(k)|$ remain same as defined in Eqns. ((6.25) to (6.27)) respectively.

Substituting the value of $s(k+1)$ in Eqn. (7.15) we get,

$$\Sigma_{i=0}^{r} C_s Z_i x_i(k+1) - \alpha' Z_i C_s x_i(k) = \{1 - q[s(k)]\} - d_s(k) + d_1.$$

Substituting the value of $x_i(k+1)$,

$$\Sigma_{i=0}^{r} Z_i C_s [F x_i(k) + G(u(k) + d(k))] - \alpha' Z_i C_s x_i(k) = \{1 - q[s(k)]\} - d_s(k) + d_1. \tag{7.16}$$

On simplifying gives,

$$\Sigma_{i=0}^{r} Z_i C_s F x_i(k) + \Sigma_{i=0}^{r} Z_i C_s G(u(k) + d(k)) - \alpha' Z_i C_s x_i(k) = \{1 - q[s(k)]\} - d_s(k) + d_1. \tag{7.17}$$

Further solving the above Eqn. (7.17) control law can be expressed as:

$$u(k) = -(C_s G')^{-1}[H x_i(k) - I x_i(k) - J(s(k)) + d_s(k) - d_1] - d(k). \tag{7.18}$$

where,

$G' = \Sigma_{i=0}^{r}(Z_i G)$, $H = \Sigma_{i=0}^{r}(C_s Z_i F)$, $I = \Sigma_{i=0}^{r} \alpha' C_s Z_i$, and $J = \{1 - q[s(k)]\}$.

This completes the $\mathbf{proof}$.

The control law defined in Eqn. (7.18) is transmitted to the network in the form of small packets. These packets will experience the controller to actuator fractional delay. Thus the mathematical model of multiple packets with controller to actuator fractional delay is given as:

$$u_a(k) = \Sigma_{j=0}^{s} D_j u_i(k - \hat{\tau}_{ca}), \tag{7.19}$$

where, $D_j = diag(0, ..., I_{s_j}, ...0)$, $u_a(k)$ is the control signal computed at actuator side and $\hat{\tau}_{ca}$ is controller to actuator random fractional delay defined using Poisson's distribution.

Using Thiran Approximation the communicated control signal in Eqn. (7.19) can be transformed to compensated control signal as,

$$u_a(k) = \Sigma_{j=0}^{s} D_j u_j(k) - \beta' D_j u_j(k-1), \tag{7.20}$$

$\beta' = \frac{\hat{\tau}_{ca}}{1+\hat{\tau}_{ca}}$.

From Eqn. (7.20), it can be noticed that at each sampling instant the effects of random packet loss and random network delay from controller to actuator are compensated in control signal through proposed technique.

The next section discusses about the stability of the closed loop system such that the system states remain within the specified band using control law (7.18).

7.4.1 Stability Analysis

The trajectories of the closed loop system (6.3,6.4) with the controller mentioned in Eqn. (7.18) in the presence of random fractional delay $(\hat{\tau})$ satisfying (6.26), multiple packet loss and matched uncertainty $d(k)$ drive towards the sliding surface (7.13) such that the following condition holds true:

$$\eta s^T(k) s(k) \succ 0. \tag{7.21}$$

Proof: The compensated sliding surface in (7.13) is given by:

$$s(k) = \Sigma_{i=0}^{r} C_i C_s x_i(k) - \alpha' Z_i C_s x_i(k-1). \tag{7.22}$$

Let us consider Lyapunov function as,

$$V_s(k) = s^T(k) s(k). \tag{7.23}$$

Taking forward difference we have,

$$\Delta V_s(k) = s^T(k+1) s(k+1) - s^T(k) s(k). \tag{7.24}$$

Using Eqn. (7.13) we get,

$$\Delta V_s(k) = [\Sigma_{i=0}^{r} Z_i C_s x_i(k+1) - \alpha' Z_i C_s x_i(k)]^T [\Sigma_{i=0}^{r} Z_i \\ C_s x_i(k) - \alpha' Z_i C_s x_i(k-1)] - \\ s^T(k)s(k). \tag{7.25}$$

Substituting the value of $x_i(k+1)$,

$$\Delta V_s(k) = [\Sigma_{i=0}^{r} Z_i C_s [F x_i(k) + G(u(k) + d(k))] - \alpha' Z_i C_s x_i(k)]^T [\Sigma_{i=0}^{r} Z_i C_s \\ [F x_i(k) + G(u(k) + d(k))] - \alpha' Z_i C_s x_i(k-1)] - \\ s^T(k)s(k). \tag{7.26}$$

Substituting the value of $u(k)$ and further simplifying it we get,

$$\Delta V_s(k) = \kappa - s^T(k)s(k). \tag{7.27}$$

where,
$\kappa = [[1 - q(s(k))]s(k) - d_s(k) + d_1]^T [[1 - q(s(k))]s(k) - d_s(k) + d_1]$. The term κ can be tuned closed to zero by appropriately selecting the parameter ψ. If κ is closed to zero then $s^T(k)s(k)$ will be larger than κ. Thus for any small parameter η we have,

$$\Delta V_s(k) \prec \eta s^T(k)s(k). \tag{7.28}$$

Thus, by tuning the parameter ψ, we have, $\Delta V_s(k) \prec \eta s^T(k)s(k)$ which guarantees the convergence of $\Delta V_s(k)$ and implies that any trajectory of the system (6.3,6.4) will be driven onto the sliding surface and maintain on it.
This completes the $\mathbf{proof}$.

7.5 Results and Discussions

In this section the efficacy of the designed control algorithm is validated in the presence of multiple packet loss and matched uncertainty applied at the input channel of the system. The simulation results are carried out using illustrative example while implementation results are carried out on DC motor plant considering various possibilities of multiple packet loss.

7.5.1 Simulation Results

Consider the continuous-time LTI system as,

$$\dot{x}(t) = Ax(t) + Bu(t - \tau_r) + Dd(t), \quad (7.29)$$

$$y(t) = Cx(t), \quad (7.30)$$

where,

$A = \begin{bmatrix} -0.7 & 2 \\ 0 & -1.5 \end{bmatrix}$, $B = \begin{bmatrix} -0.03 \\ -1 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Dsicretizing the above system with sampling interval of $h = 30msec$ we get,

$$x(k+1) = Fx(k) + Gu(k - \hat{\tau}) + d(k), \quad (7.31)$$

$$y(k) = Cx(k), \quad (7.32)$$

where,

$F = \begin{bmatrix} 0.9792 & 0.05805 \\ 0 & 0.956 \end{bmatrix}$, $G = \begin{bmatrix} -0.001771 \\ -0.02934 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

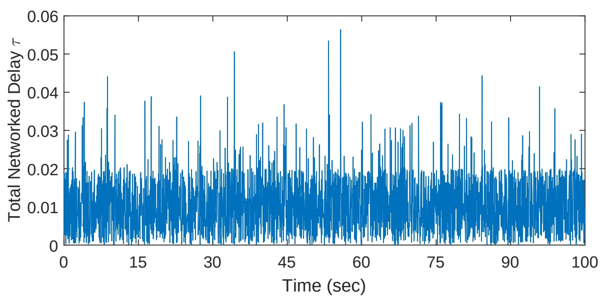


Figure 7.3: Total networked fractional delay

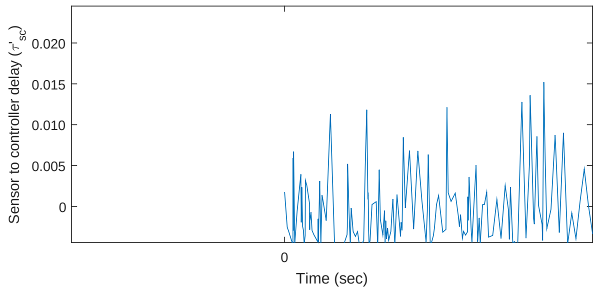


Figure 7.4: Magnified sensor to controller random fractional delay

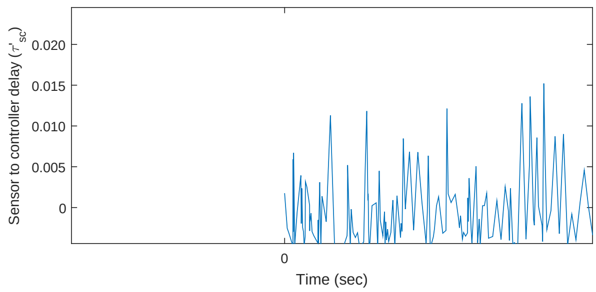


Figure 7.5: Magnified controller to actuator random fractional delay

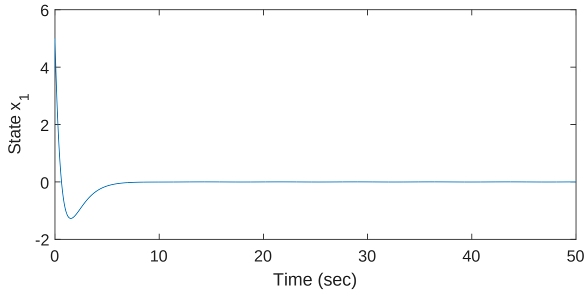


Figure 7.6: State variable $x_1(k)$

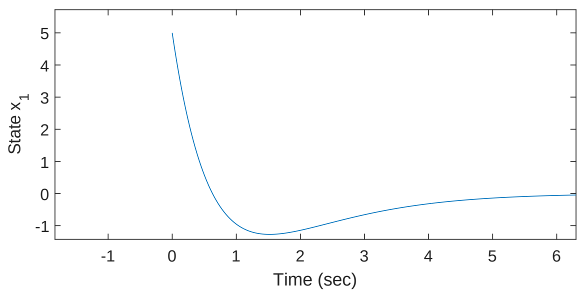


Figure 7.7: Magnified state variable $x_1(k)$

Figures (7.6) to (7.21) shows the nature of the system in the presence of network non-idealities. In order to prove the robustness of the proposed control algorithm slow time varying disturbance is applied to the input of the system. Figure (7.3) shows the response of total network induced delay modelled using Poisson's distribution. It is assumed that the probability of the networked delay lesser than sampling interval is

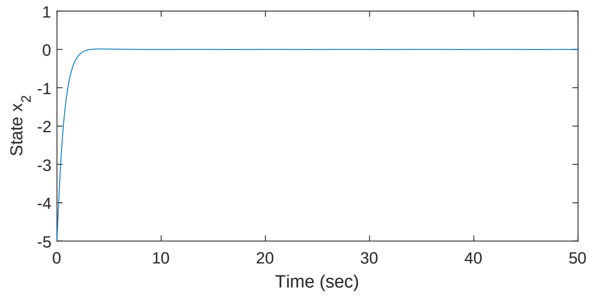


Figure 7.8: State variable $x_2(k)$

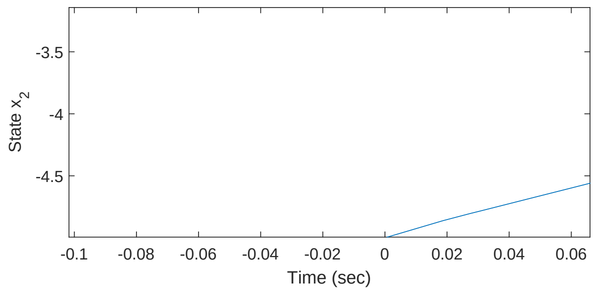


Figure 7.9: Magnified state variable $x_2(k)$

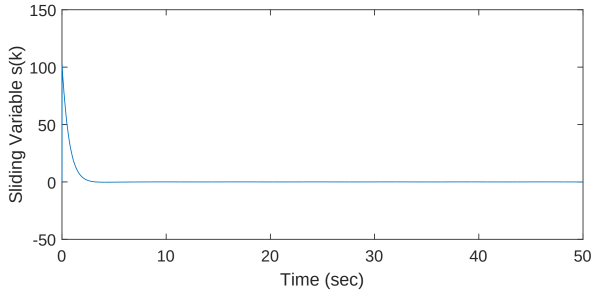


Figure 7.10: Compensated sliding variable $s(k)$

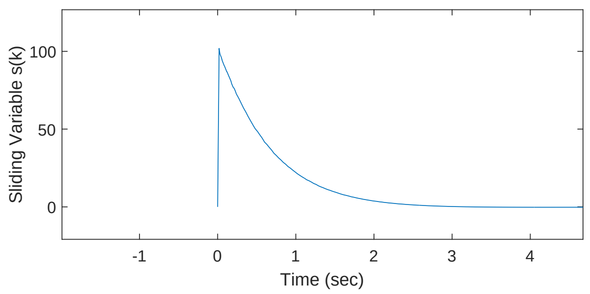


Figure 7.11: Magnified compensated sliding variable $s(k)$

$p = 0.67$ while the probability of networked delay greater than sampling interval is $p = 0.33$. This assumption indicates that according to Poisson's distribution at every sampling interval at least one event is generated with zero trial. Thus, the specified network delay range computed through Poisson's distribution is $0.003sec \leq \tau \leq 0.055sec$ respectively.

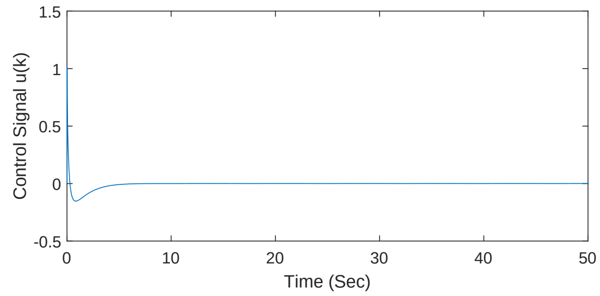


Figure 7.12: Control signal $u(k)$

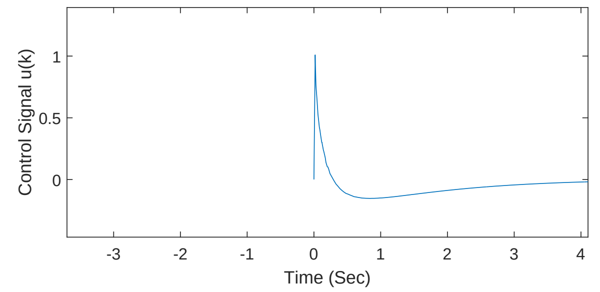


Figure 7.13: Magnified control signal $u(k)$

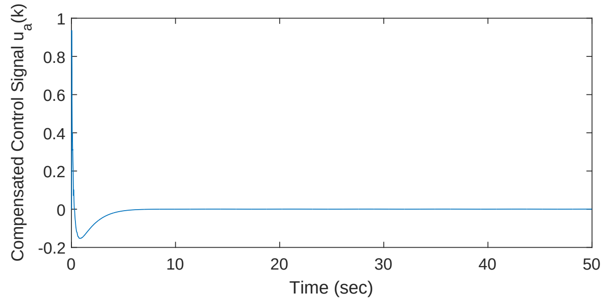


Figure 7.14: Compensated control signal $u_a(k)$

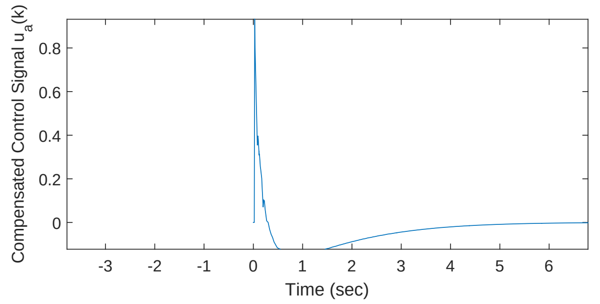


Figure 7.15: Magnified compensated control signal $u_a(k)$

The sliding gain parameter computed using discrete LQR method with $Q = diag(1000, 1000)$ and $R = 1$ is $C_s = [-14.234 \quad -20.625]$ having $\psi = 10$. The quasi-sliding mode band comes out to be $|s(k)| \preceq +0.1$ to -0.1.

Figures (7.6) to (7.15) show the nature of state variables, sliding surface, control signal computed at controller side and compensated control signal under multiple packet trans-

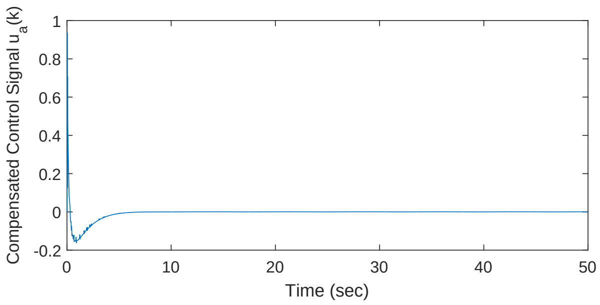


Figure 7.16: Compensated control signal $u_a(k)$ with 10$\%$ packet loss

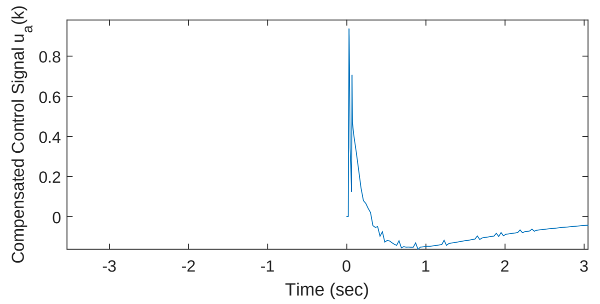


Figure 7.17: Magnified compensated control signal $u_a(k)$ with 10$\%$ packet loss

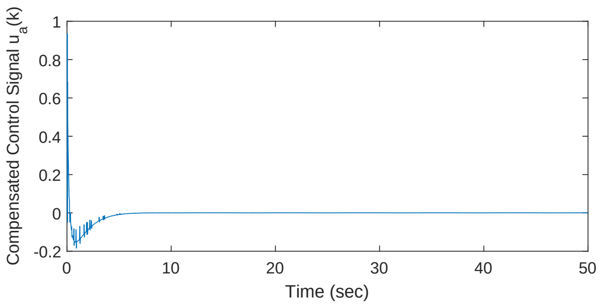


Figure 7.18: Compensated control signal $u_a(k)$ with 20$\%$ packet loss

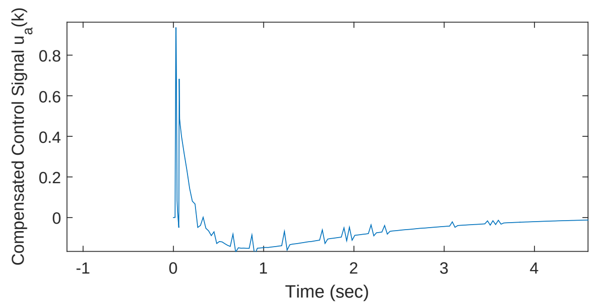


Figure 7.19: Magnified compensated control signal $u_a(k)$ with 20$\%$ packet loss

mission. Figures (7.6) and (7.8) show the nature of state variables $[x_1 \quad x_2]$ with initial conditions $[5 \quad -5]$ respectively. It is observed that both the state variables converge to zero from their specified initial condition in the presence of specified random fractional delay and multiple packets transmission. In order to determine the exact effect of random fractional delay the magnified results of the same are shown in Figures (7.7)

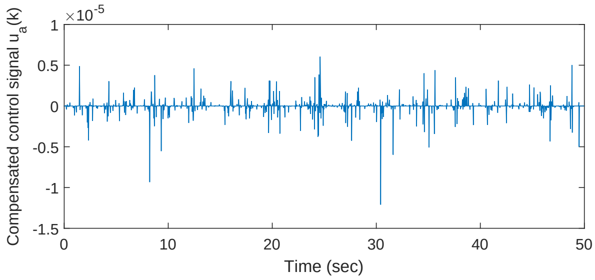


Figure 7.20: Compensated control signal $u_a(k)$ with 30% packet loss

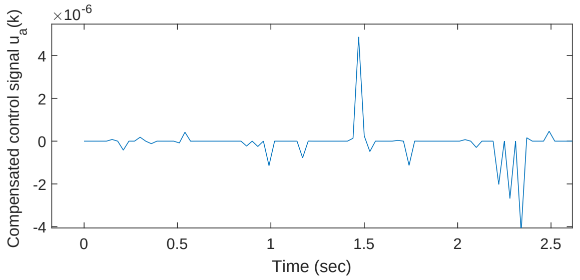


Figure 7.21: Magnified compensated control signal $u_a(k)$ with 30% packet loss

and (7.9) respectively. It is noticed that both the state variables are computed from first sampling instant even in the presence of random fractional delay and multiple packet transmission in forward and feedback channel. The same effect of fractional delay compensation is observed in sliding variable Figure (7.10) and control signal Figure (7.12) computed at controller side. It is observed that both the parameters are computed from first sampling instant even in the presence of sensor to controller random fractional delay as shown in Figure (7.4). The magnified results of sliding variable and control signal computed at controller side are shown in Figures (7.11) and (7.13) respectively. Figure (7.14) shows the result of compensated control signal computed at the actuator end. Observing the magnified result in Figure (7.15), it is noticed that the effect of random fractional delay from controller to actuator is also compensated as it is computed from first sampling instant. Figure (7.5) shows the magnified result of controller to actuator random fractional delay.

The robustness of the proposed algorithm is further extended for multiple packet loss. The multiple packet loss is considered in both the channels simultaneously. It is assumed that more than one packet is lost on either side of the channel during transmission. Figures (7.16), (7.18) and (7.20) show the nature of compensated control signal computed at actuator side under 10%, 20% and 30% multiple packet loss respectively. It can be observed that the effect of random fractional delay from controller to actuator is compensated precisely as the controller signal is computed from first sampling instant

in the case of 10% and 20% packet loss. While in 30% packet loss the effect of random fractional delay is not compensated. The magnified results of the same are shown in Figures (7.17), (7.19) and (7.21) respectively.
Thus, the proposed control algorithm compensates the effect of random fractional delay and shows the stable response satisfying Eqn. (7.21) in the presence of multiple packet loss and matched uncertainty.

7.5.2 Experimetal Results

In this section the proposed control algorithm is validated on Quanser DC Motor as a plant connected in networked medium. The position of the motor is controlled for the given reference input in the presence of mutiple packet transmission and matched uncertainty. The state space form of DC Motor plant of Eqn. (3.34) is given as,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau_r) + Dd(t), \tag{7.33}$$

$$y(t) = Cx(t), \tag{7.34}$$

where,
$A = \begin{bmatrix} -201 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Dsicretizing the above system with sampling interval of $h = 30msec$ we get,

$$x(k+1) = Fx(k) + Gu(k-\hat{\tau}) + d(k), \tag{7.35}$$

$$y(k) = Cx(k), \tag{7.36}$$

where,
$F = \begin{bmatrix} 0.001836 & 0 \\ 0.004573 & 1 \end{bmatrix}$, $G = \begin{bmatrix} -0.004753 \\ -0.0001242 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

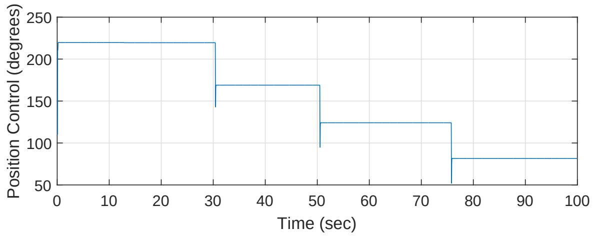


Figure 7.22: Position Control of DC Motor under multiple packet transmission

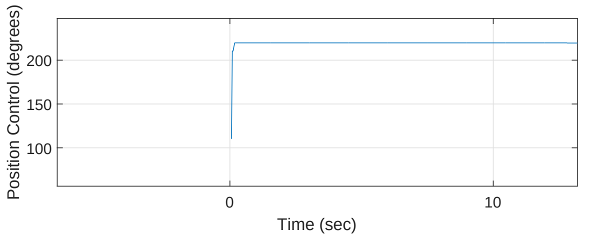


Figure 7.23: Magnified position control

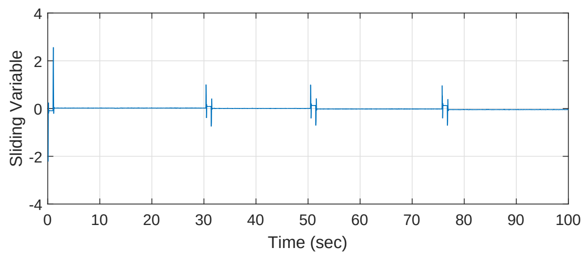


Figure 7.24: Compensated sliding variable $s(k)$

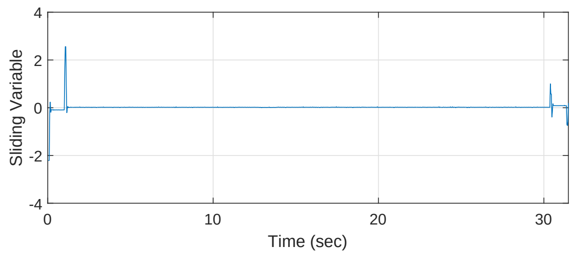


Figure 7.25: Magnified compensated sliding variable $s(k)$

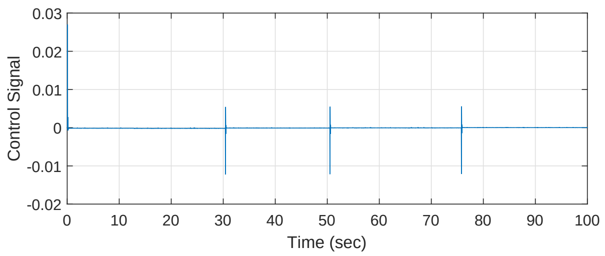


Figure 7.26: Control signal $u(k)$

Figures (7.22) to (7.35) show the nature of DC motor plant in the presence of random network fractional delay, multiple packet loss and matched uncertainty. The values of sliding gain C_s, sliding band $|s(k)|$ and user defined constant ψ are same as

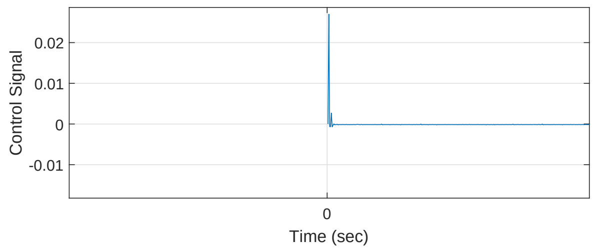


Figure 7.27: Magnified control signal $u(k)$

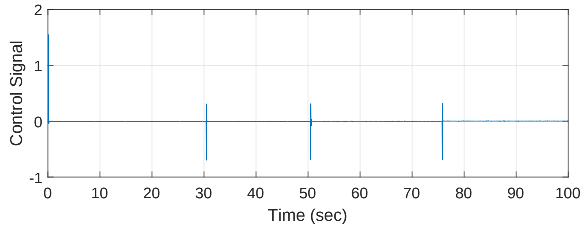


Figure 7.28: Compensated control signal $u_a(k)$

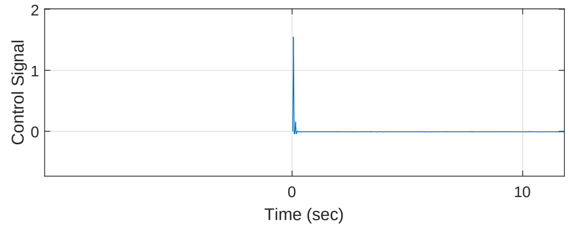


Figure 7.29: Magnified compensated control signal $u_a(k)$

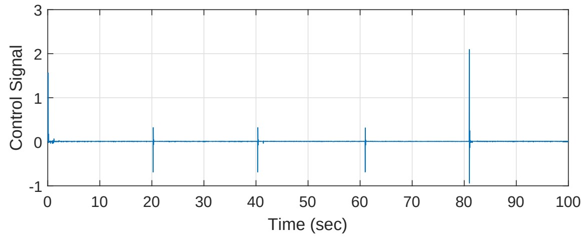


Figure 7.30: Compensated control signal $u_a(k)$ with 10% multiple packet losses

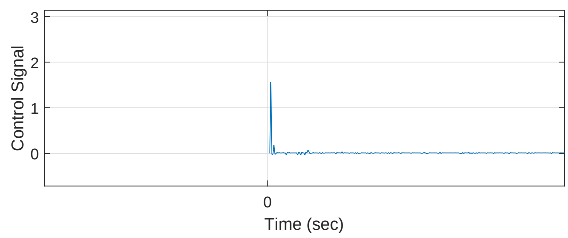


Figure 7.31: Magnified compensated control signal $u_a(k)$ with 10% multiple packet loss

considered in simulation section. The results of position control, sliding variable, control signal computed at the controller side and compensated control signal computed at actuator side are carried out in the presence of specified network delay range of

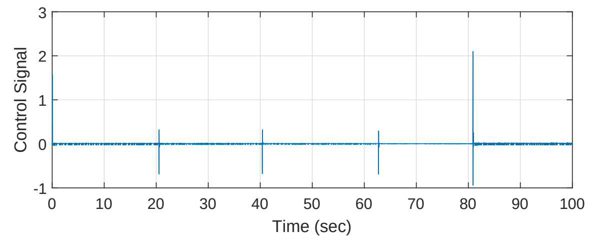


Figure 7.32: Compensated control signal $u_a(k)$ with 20$\%$ multiple packet losses

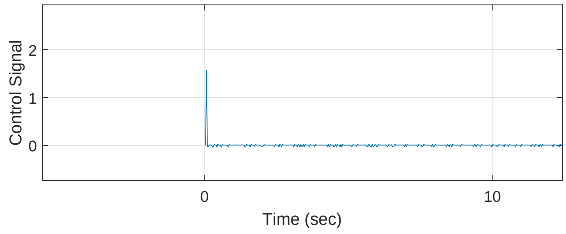


Figure 7.33: Magnified compensated control signal $u_a(k)$ with 20$\%$ multiple packet losses

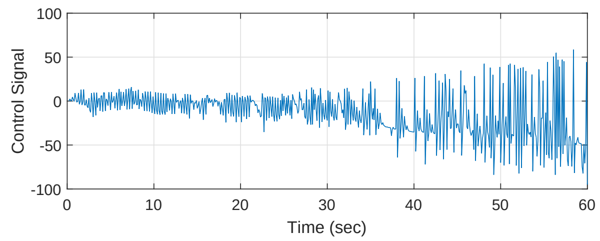


Figure 7.34: Compensated control signal $u_a(k)$ with 30$\%$ multiple packet losses

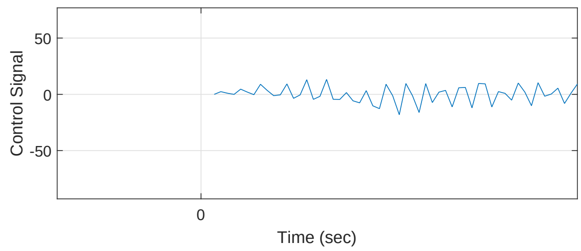


Figure 7.35: Magnified compensated control signal $u_a(k)$ with 30$\%$ multiple packet losses

$0.003sec \leq \tau \leq 0.055sec$. This delay range is computed through Poisson's distribution under same assumptions mentioned in simulation section. The slow time varying disturbance is applied to the input of the system defined as matched uncertainty.

The implementation results are carried out in two parts: (i) the first part (Figures (7.22) to (7.29)) describes the effect of compensation algorithm on the system under multiple packet transmission, (ii) while second part (Figures (7.30) to (7.35)) specifies the effect of random fractional delay on the control signal computed at the actuator side in the presence of multiple packet loss.

Figure (7.22) shows the nature of reference tracking response of DC motor plant in the presence of network non-idealities. It is noticed that the position of the DC motor is controlled according to change in the reference signal for the specified fractional delay and matched uncertainty under multiple packet transmission. In order to show the exact effect of compensation the magnified result of the same is shown in Figure (7.23). It is observed that the output signal responds the reference input at first sampling instant even in the presence of random fractional delay. The same effect of compensation algorithm is observed in sliding surface (Figure (7.24)), control signal computed at the controller side (Figure (7.26)) and compensated control signal computed at the actuator side (Figure (7.28)). It can be observed that all the three parameters slides towards the origin and remain within the specified band (6.27) over a finite interval of time. The magnified results of the same are shown in Figures (7.25), (7.27) and (7.29) respectively. Observing the magnified results it is noticed that all the three parameters are computed from initial sampling instant even in the presence of sensor to controller fractional delay and controller to actuator fractional delay.

Figures (7.30) to (7.35) show the nature of compensated control signal computed at actuator side under multiple packet loss. It is assumed that multiple packet loss takes place on either side of the network medium. So the actual effect of multiple packet loss can be studied at the actuator side rather than controller side. Figures (7.30), (7.32) and (7.34) shows the nature of compensated control signal at the actuator side with packet loss probabilities of 10%, 20% and 30% respectively. It is noticed that as the multiple packet loss probability increases the robust terms will generate more control actions to stabilize the system which in turn makes the system oscillatory in nature. The control signal shows the unacceptable response when the packet loss probability increases to 30%. Moreover, it is also observed from magnified results ((7.31), (7.33) and (7.35)) that the effect random fractional delay within the network is also not compensated as the packet loss is increased.

Thus, from results it can be noticed that with real time system the proposed algorithm shows the stable response satisfying (7.21) under 20% multiple packet loss for the specified random fractional delay and matched uncertainty.

7.6 Conclusion

In this chapter, time delay approximation algorithm is proposed in discrete-time domain that compensates the effect of variable fractional delay in the presence of mutiple packet loss condition. The random fractional delay is modelled using Poisson's distribution while multiple packet loss is modelled using probability distribution function. The multiple packet loss model is derived at the controller side as well as at the actuator side. The effects of random fractional delay as well as multiple packet loss are compensated in forward channel as well as feedback channel. The discrete-time sliding mode controller is designed based on the proposed sliding surface. The stability condition of the closed loop system is derived using Lyapunov approach such that the system states remain within the specified band over a finite interval of time. The efficacy of the proposed control algorithm is checked under different conditions through illustrative example and real-time plant. The simulation and experimental results shows that the discrete-time control law derived using Thiran's Approximation technique compensates the effect of random fractional delay even in the presence of multiple packet loss with probability of 20% and matched uncertainty.

CHAPTER 8

CONCLUSION, FUTURE SCOPE AND CHALLENGES

8.1 Conclusion and Future Scope

In this thesis, a novel idea of compensating the fractional delay in the sliding surface is introduced. The effect of fractional delay generated in NCS due to the presence of the communication medium is compensated using Thiran approximation technique. The sliding surface designed so that it slides along the predetermined surface according to fractional delays. Using this novel approach, a switching-type discrete-time networked sliding mode controller is designed which computes the control sequences in the presence of deterministic fractional delay and matched uncertainty. The stability of the closed loop system is assured by using Lyapunov approach. The efficacy of the proposed algorithm is tested on DC Servo motor setup with different network delays and external disturbances. The results were also compared with the conventional SMC. It is concluded that the fractional delay approximated using Thiran approximation is more efficient technique as it compensates the fractional networked delays then conventional discrete-time sliding mode control.

The major drawback of switching type discrete-time sliding mode controller is that it generates more chattering due to which the system generates oscillatory type of behaviour. So, to overcome this issue a non-switching type discrete-time sliding mode controller is designed such that system states slide along the proposed compensated surface and maintain within the specified band. The stability of the closed loop system is assured using Lyapunov approach through proposed control law. The efficacy of the proposed algorithm is tested through illustrative example as well as DC servo motor setup with different deterministic fractional delays and matched uncertainty. The results were also compared with switching type SMC as well as conventional SMC. It is concluded that non-switching type SMC provides faster convergence without increasing the amplitude of control signal and offers better fractional delay compensation than switching-type SMC and conventional SMC. The efficacy of the proposed control algorithm is also tested under real-time networks using True-Time simulator. The performance of the control algorithm is checked using CAN and Switched Ethernet as a network medium in the presence of packet loss condition. It can be noticed from results that the control law derived using Thiran's Approximation compensates the effect of fractional network delay very precisely even in the presence of real-time networks and packet loss situation.

Later on, the concept of Thiran Approximation is used in Multi-rate Output feedback approach in which the sliding surface and control law are computed based on availability of the output information. The main advantage of using multirate output feedback approach is that the system states are computed based on the output information available and the error between computed as well as estimated state variables becomes zero exactly after one sampling instant. Using this novel approach a multirate output feedback discrete-time networked sliding mode control law is derived that compute the control sequences in the presence of deterministic fractional delay and matched uncertainty. The stability condition of closed loop NCSs is derived using Lyapunov approach that ensures finite time convergence of system states in presence of network non-idealities. The effectiveness of the proposed algorithm is examined under different possible conditions through illustrative example.

Further, the concept of Thiran Approximation is examined for random fractional delays with single packet loss situation. The random fractional delay is modelled using Poisson's distribution function and packet loss is modelled using Probability distribution function. The discrete-time sliding mode controller is designed using compensated sliding surface in the presence of random fractional delays, packet loss and matched uncertainty. The stability of the closed loop system is derived that ensures finite time convergence in the presence of random fractional delay, packet loss and matched uncertainty. The effectiveness of the proposed control algorithm is examined through DC servo motor setup under random fractional delay and packet loss. The results proved that the control law derived using Thiran's Approximation compensates random fractional delay accurately even in the presence of single packet loss with probability of 30% as well as networked delays having values greater than sampling interval.

Lastly, the same concept of Thiran Approximation is carried out for random fractional delay with multiple packet loss situations. The multiple packet loss situation is modelled using Probability distribution function while random fractional delay is modelled using Poisson's distribution. The discrete-time sliding mode control law is derived that compensates the effect of random fractional delay with multiple packet loss using compensated sliding surface. The stability of the closed loop NCS is assured through Lyapunov approach under multiple packet transmission. The efficiency of proposed control algorithm is verified through DC servo motor plant under random fractional delay, multiple packet loss and matched uncertainty. The results proved that the proposed control law works efficiently and compensates the effect of random fractional delay even in the presence of multiple packet loss with probability of 30% as well as networked delays greater than sampling interval.

In future, the proposed control algorithm can be extended for Wireless Networked Control System (WNCS) as it possesses random time delay and packet loss. Morover the efficacy of the proposed algorithm is checked for direct structure NCS. The same can be extended for hierarchical structure and shared network structure NCS. The results proved that proposed control algorithm operates properly for the state feedback based controller in all different situations such as deterministic delay, random delay, single packet loss and multiple packet loss. In future, the same work can be extended for multi-rate output feedback controllers considering the same network non-idealities.

8.2 Challenges

In Networked Control System although much work has been carried out, but still there are various challenges that has to be taken care while designing any control algorithm. NCS deals with various open research problems due to the presence of communication medium. Some of the challenges are:

- In some of the applications it might be possible that the delays are greater than the sampling interval. So, in such cases the control algorithm designed for time delay lesser than sampling interval will not able to work efficiently. Thus, in such cases there is a need for developing some robust technique which takes care of the networked delay having value greater than sampling interval.
- There are no standard algorithm available in the literatures that discusses about the variable packet loss model. However, in the existing models, network-induced delays and packet dropouts are lumped together. As a result, it is hard to distin-

guish their respective effects on NCSs. Thus, there is need for developing some model that takes care of only packet loss.

- In event-triggered control and filtering, it is often to assume that packet dropouts and packet disorders do not occur. This assumption is not practical when packets are transmitted through a communication network. How to deal with packet dropouts and packet disorders in event triggered control and filtering is challenging. Up to date, taking packet dropouts and packet disorders into account, no efficient approaches are proposed to design event triggered controllers and filters.

- Distributed networked control and distributed network based filtering are still attractive and challenging. Although some results on these issues are reported in the literature, they are usually based on some strong assumptions when parts of network constraints are taken into account. In fact, many practical factors need to be considered for distributed control and filtering in network environments. To mention a few, network-induced delays in different channels between an agent and its neighbours are different and time-varying, packet dropouts occur randomly in different communication channels, the topologies of the agents in a distributed system are not always the same at any time, and so on. These factors indeed make the analysis and synthesis of distributed networked control and distributed network-based filtering more complicated, especially for distributed systems with a large number of agents. (Zhang *et al.*, 2016)

- For industrial control and applications of NCSs, it is interesting to investigate possible positive effects of network-induced delays and packet dropouts on NCSs. There is no such algorithm designed that considers the positive effect of these parameters for industrial applications.

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PATENT, PUBLICATIONS AND ACKNOWLEDGEMENTS

- **Indian Patent**

1. **A J Mehta**, **D H Shah**, **H A Mehta** and **R D Shah**, *Discrete-Time Sliding Mode Controller for Networked Control System with Random Fractional Delay*, **Application No. 201721015486**, May, 2017.

- **Book**

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